BONNEVILLE POWER ADMINISTRATION

Regression for M&V: Reference Guide

July 2018

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Version 2.0

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Prepared for

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Table of Contents

1. Introduction

1.1. Purpose

Regression for M&V: Reference Guide (Regression Guide) as a complement to the Measurement and Verification (M&V) protocols used by the Bonneville Power Administration (BPA). It assists the engineer in conducting regression analysis to control for the effects of changing conditions (such as weather) on energy consumption.

Originally developed in 2012, this *Regression Guide* is one of ten documents produced by BPA to direct M&V activities; an overview of the ten documents is given in the *Measurement and Verification (M&V) Protocol Selection Guide and Example M&V Plan (Selection Guide).*

Chapter 8 of this guide provides full citations (and web locations, where applicable) of documents referenced and an appendix provides a glossary specific to this guide.

1.2. Protocols Version 2.0

BPA revised the protocols described in this guide in 2018. BPA published the original documents in 2012 as Version 1.0. The current guides are Version 2.0.

1.3. How is M&V Defined?

BPA's *Implementation Manual* (the IM) defines measurement and verification as "the process for quantifying savings delivered by an energy conservation measure (ECM) to demonstrate how much energy use was avoided. It enables the savings to be isolated and fairly evaluated."[1] The IM describes how M&V fits into the various activities it undertakes to "ensure the reliability of its energy savings achievements." The IM also states:

> *The Power Act specifically calls on BPA to pursue cost-effective energy efficiency that is "reliable and available at the time it is needed."*[2] *[…] Reliability varies by savings type: UES, custom projects and calculators.*[3,4] *Custom projects require site-specific Measurement and Verification (M&V) to support reliable estimates of savings. BPA*

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1 2017-2019 Implementation Manual, BPA, October 1, 2017. *https://www.bpa.gov/EE/Policy/IManual/Documents/IM_2017_10-11-17.pdf*

2 Power Act language summarized by BPA.

3 UES stands for Unit Energy Savings and is discussed subsequently. In brief, it is a stipulated savings value that region's program administrators have agreed to use for measures whose savings do not vary by site (for sites within a defined population). More specifically UES are specified by either the Regional Technical Forum – RTF (referred to as "RTF approved") or unilaterally by BPA (referred to as BPA-Qualified). Similarly, Savings Calculators are RTF approved or BPA-Qualified.

4 Calculators estimate savings that are a simple function of a single parameter, such as operating hours or run time.

M&V Protocols direct M&V activities and are the reference documents for reliable M&V. For UES measures and Savings Calculators, measure specification and savings estimates must be RTF approved or BPA-Qualified.[5]

The *Selection Guide* includes a flow chart providing a decision tree for selecting the M&V protocol appropriate to a given custom project and addressing prescriptive projects using UES estimates and Savings Calculators.

M&V is site-specific and required for stand-alone custom projects. BPA's customers submit bundled custom projects (projects of similar measures conducted at multiple facilities) as either an M&V Custom Program or as an Evaluation Custom Program; the latter requires evaluation rather than the site-specific M&V that these protocols address.

1.4. Background

BPA contracted with a team led by kW Engineering, Inc. to assist the organization in revising the M&V protocols that were published in 2012 and used to assure reliable energy savings for the custom projects it accepts from its utility customers. The team conducted a detailed review and user assessment of the 2012 M&V Protocols and developed the revised version 2.0 under Contract Number 00077045.

The kW Engineering team is comprised of:

- kW Engineering, Inc. (kW), led by David Jump, Ph.D., PE, CMVP
- Research into Action (RIA), led by Marjorie McRae, Ph.D.
- Demand Side Analytics (DSA), led by Jesse Smith

BPA's Todd Amundson, PE and CMVP, was project manager for the M&V protocol update work. The kW Engineering team compiled feedback from BPA and regional stakeholders, and the team's own review to revise and update this 2018 *Regression Guide*.[6] The kW Engineering team would also like to thank Gregory Brown of Brolte, LLC and Josh Rushton of Rushton Analytics for their input.

[5] https://www.bpa.gov/EE/Policy/IManual/Documents/IM_2017_10-11-17.pdf, page 1.

[6] William Koran, formerly of QuEST, was the primary author of Version 1.0 of the Regression Guide, under Todd Amundson's direction and supported by other members of the protocol development team.

2. Overview of Regression

2.1. Description

Regression is a statistical technique that estimates the dependence of a variable of interest (such as energy consumption) on one or more independent variables, such as ambient temperature. A regression model estimates the effects on the dependent variable of changes in a given independent variable, controlling for the influence of other variables. It is a powerful and flexible technique that can be used in a variety of ways when measuring and verifying the impact of energy efficiency projects.

This protocol assumes the use of ordinary least squares (OLS) regression. OLS is the most common form of regression modeling and the default approach in most software packages. OLS is a mathematical procedure to solve for the set of coefficients that minimize the sum of the squared differences between the raw data and the fitted linear trend. There are many other forms of regression modeling, but they are outside the scope of this protocol.

These guidelines are intended to provide energy engineers and M&V practitioners with a basic understanding of the relevant statistical measures and assumptions necessary to properly use regression analysis. The guidelines should be followed whenever the technique is required. While this is not a comprehensive guide to regression, following the approaches described here should make most M&V regressions valid for their intended purposes. Please refer to a textbook for more comprehensive information.

Many sources offer additional information on regression analysis. Resources that may be valuable references for energy efficiency M&V practitioners engaged in regression modeling include the following:

- *IPMVP: International Performance Measurement and Verification Protocol: Concepts and Options for Determining Energy and Water Savings, Volume 1* (IPMVP, 2012) and *Core Concepts* (IPMVP, 2016)[7]
- *ASHRAE Guideline 14-2014 – Measurement of Energy, Demand, and Water Savings*[8]
- *California Commissioning Collaborative's Guidelines for Verifying Existing Building Commissioning Project Savings, Using Interval Data Energy Models: IPMVP Options B and C*.[9]

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7 See especially *Uncertainty Assessment, IPMVP*.

8 *Annex B*, *Determination of Savings Uncertainty*, and *Annex D, Regression Techniques*, have information very relevant to regression analysis.

9 This is a relatively easy-to-read document that focuses on regression methods. Although written with a focus on commissioning of existing buildings, the methods described are applicable to a variety of projects.

In addition to these documents, a general reference for exploratory data analysis and statistical inference, the *NIST/SEMATECH Engineering Statistics Handbook*, is available online from the National Institute of Standards and Technology. The *Engineering Statistics Handbook* site includes a detailed table of contents for the web-based handbook, and also includes downloadable PDF files for off-line reading.

2.2. Regression Applicability

Regression estimation is useful when a simple spot measurement is not adequate to establish the baseline energy use. It is applicable when the energy use affected by the efficiency measure is correlated to one or more independent variables. Note that the technique of energy indexing is a simple application of the regression guide that can be used when energy use is linearly proportional to one normalizing (independent) variable. There are other constraints upon using energy indexing in lieu of a more generalized approach. Please refer to BPA's *Verification by Energy Use Indexing Protocol* for further information on this technique.

In M&V, energy usage is typically (and optimally) the dependent variable, whether energy usage is measured monthly through bills or measured more frequently through meter monitoring. The regression model attempts to predict the value of the dependent variable based on the values of independent, or explanatory, variables such as weather data.

- **Dependent Variable** – the outcome or endogenous variable; the variable described by the model; for M&V, the dependent variable is typically energy use
- **Independent Variable** – an explanatory or exogenous variable; a variable whose variation explains variation in the outcome variable; for M&V, weather characteristics are often among the independent variables
- **Simple Regression** – a regression with a single independent variable
- **Multiple Regression** – a regression with two or more independent variables

One of the most common applications of regression in M&V is to understand the factors that influence monthly utility consumption. The initial step is to establish the baseline dependence of building energy usage on weather conditions and other independent variables (for example, occupancy and production) by modeling the period prior to the retrofit that is illustrative of pre-retrofit usage – the baseline period. Then, post-retrofit independent variables are applied to the baseline model to estimate the building's energy use had the energy efficiency improvements not been made (the *counterfactual situation*). In M&V, this projection of the baseline energy use into the post period is typically called the *adjusted baseline*. Finally, the adjusted baseline (predicted counterfactual energy use) is compared to the actual post-retrofit energy use and the difference provides an estimate of energy savings.[10]

[10] Note that this is the general approach followed by most M&V practitioners to estimate energy savings. Economists, who typically conduct impact evaluations, typically estimate a single model from both baseline and post-retrofit data, and use a dummy (*categorical*) variable applied to post-retrofit observations to

When available, the practitioner can use more granular independent variable data to model energy with a much smaller time interval than a monthly billing period, such as hourly, daily or weekly data. These smaller interval data are frequently applicable to *IPMVP Options A (Key Parameter Measurement), B (All Parameter Measurement)*, and *C (Whole Facility)*, and can also be used to assist in model calibration for *IPMVP Option D* (*Calibrated Simulation*).

2.3. Advantages of Regression

Regression is a very flexible technique that can be used in conjunction with other M&V methods to help provide a deeper understanding of how and when energy is used. The ideal case for regression is when the measurement period captures the full annual variation in the dependent and independent variables – that is, the full range of operation conditions. If the relationship between the independent and dependent variables is not expected to change over the range of operating conditions, then short-term measurements can be extrapolated to annual energy use, even if the measurement period does not capture the full annual variation.

Regression not only facilitates an estimate of energy savings, but also can provide an estimate of the uncertainty in savings calculations. Further, a baseline regression model can be used to estimate how much data is required in the post-retrofit period to keep savings uncertainty below a desired threshold.

Regression is conceptually simple. Most M&V practitioners have at least a basic familiarity with regression analysis. Further, usage and weather data – the variables typically needed for a basic model – usually are readily available.

2.4. Disadvantages of Regression

Although simple in concept, proper use of regression requires a clear understanding of statistical methods and application guidance, which this document seeks to provide to the M&V practitioner. The information in this guide should be relevant to most M&V projects, but situations can occur that require a more detailed understanding of statistical methods. While the basic technique is straightforward, complications to the site or the data can easily require more advanced techniques and a more thorough understanding of regression methods than this document can provide.

Regression models require multiple observations on the dependent and independent/explanatory variables. There are times, however, when explanatory variables are not readily available, or we only have access to proxies. Explanatory variables omitted from a regression model typically introduce error. If energy use is not a strong function of the independent variable(s) in the equation, or if there is large variability in energy use relative to strength of the predictive relationship ("scatter" in the *x-y* chart; discussed in Section 3.3 and 3.4), regression analysis generates estimates that have high uncertainty.

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estimate energy use savings. The resulting savings estimates are comparable to the approach described here, although not necessarily identical.

2.5. Consider Uncertainty when Choosing to Use Regression Analysis

The relative precision – or fractional savings uncertainty (FSU) – of an energy savings estimate is the magnitude of the uncertainty relative to the estimate of annual savings.[11] (Note that the savings need not be annual – perhaps they are just the savings achieved through the reporting period to date. The formulas just need to be used accordingly.) If a project is expected to save 300,000 kWh per year and the uncertainty – or margin of error – is ± 75,000 kWh/year, the relative precision of the estimate is ± 25%. The Verification by Energy Modeling and Verification by Energy Use Indexing protocols include guidance for calculating the expected uncertainty using baseline data. The key drivers of relative precision are:

1. **The size of the signal** – it is easier to precisely measure large effects than small effects. Savings uncertainty is not a function of expected savings, but the ability of the model to explain variation in observed usage.

2. **Amount of noise in the data** – it is easier to precisely measure effects when much of the variation in pre-installation energy consumption is explained by known independent variables like weather or production. Savings from projects in facilities with noisy, or erratic, load patterns will be more uncertain than projects in facilities with more predictable load patterns.

3. **The frequency of the data** – it is easier to precisely measure effects with daily or hourly energy usage data than with monthly data. However, because autocorrelation distorts the information coming from traditional statistical calculations, the practitioner using hourly or sub-hourly data will need to take additional modeling steps to produce unbiased estimates of uncertainty than is necessary with lower frequency data.

The larger the signal, the less the noise, and the higher the frequency of the data each increase the likelihood that Energy Modeling or Energy Indexing will be an appropriate M&V approach.

Practitioners should be mindful of relative precision before selecting a regression-based protocol. If the expected relative precision (or FSU) of a project savings estimate is greater than ± 50%, an alternative protocol could be more appropriate. It is common to find uncomfortably high relative precision estimates for projects expected to save less than 5% of facility energy use. End-use metering may be able to isolate the affected end-use(s) within a facility and significantly reduce the amount of noise in the data being modeled (without changing the size of the expected effect).

[11] ASHRAE. 2014. *ASHRAE Guideline 14-2014 – Measurement of Energy, Demand, and Water Savings*. Atlanta, Ga.: American Society of Heating, Refrigerating and Air-Conditioning Engineers. https://www.techstreet.com/standards/guideline-14-2014-measurement-of-energy-demand-and-water-savings?product_id=1888937, sec. B4, p. 88 describes fractional savings uncertainty (FSU).

3. The Regression Process

The regression process can be summarized in seven steps, discussed in detail in the following sections:

1. Identify all independent variables to be included in the regression model
2. Collect the data
3. Clean the data
4. Graph the data
5. Select and develop the regression model
6. Validate the model
7. Analyze the residuals of the model[12]

Note that for ordinary linear regression to be an appropriate analysis method, the following conditions must be met:

1. The modeler should be able to reasonably explain the relationship between the dependent and independent variable(s) prior to performing any regression analysis.
2. The relationship between the dependent variable and all independent variables in the model (except for indicator variables)[13] should be approximately linear.
3. The model residuals must follow an approximate Normal distribution with a mean of zero and a constant variance (a condition termed homoscedasticity).
4. The model residuals model must be uncorrelated with each of the independent variables in the regression model.
5. The model residuals must be independent. That is, the residual at time t must not be correlated with the residual at time $t - 1$ or at any other time period.

Condition 2, self-explanatory for linear regression, is nonetheless discussed in slightly more detail in Section 3.4; conditions 3-5 are discussed in greater detail in Section 3.7 and Section 5.4.

[12] Model residuals are the differences between the actual and predicted values.

[13] An indicator variable is a binary variable that indicates the presence or absence of some categorical condition expected to shift the outcome.

3.1. Step 1 - Identify All Independent Variables

To properly identify all independent variables, you should consider the facility and how different factors play into its energy use. Then, you will compile a list of the variables that are likely to have an impact on the energy use of the facility or system being modeled. When independent variable values are not numeric or are not continuous, the data can be separated into several regression models, rather than including all variables within a single model. For example, separate regression models may be developed for a food processing facility with distinct on- and off-season production operating modes, resulting in better estimation of baseline energy usage compared to a single model.

Developing separate models is just one approach to working with categorical variables, an approach favored by many M&V practitioners. One can also use *binary* variables to indicate the presence or absence of a given condition (that is, to create a category) and apply these binary variables to develop estimates of either the slope or the intercept, or both, when the given condition is satisfied. (See Section 4.4.1 for a discussion of the use of categorical variables.)

We advise caution when including many variables. A model should only use the variables that explain the relationship and not include additional, extraneous information. *ASHRAE Guideline 14, Appendix D*, provides additional information on regression estimation with two or more independent variables (*multiple regression*).

Some independent variables commonly used in energy regressions are:

- Ambient dry bulb temperature (actual or averaged over a time-period such as a day)
- Heating degree-days (HDD: See Section 6.2)
- Cooling degree-days (CDD: See Section 6.2)
- Plant output (number of widgets produced in some period)
- Number of occupants in a facility each hour

3.2. Step 2 - Collect Data

Prior to installation of the measure, identify and collect data for a monitoring period that is representative of the facility, operation, or equipment. This is the *baseline* period, sometimes referred to as the *tuning* or *pre*-period. To provide accurate predictions, the sample of data used to estimate a regression model should be representative of the full range of operating conditions. That is, the baseline monitoring period should be long enough to provide “coverage” of the full range of operating conditions. For example, when analyzing savings for a weather-sensitive measure, the baseline period typically includes 12 or 24 months of consumption data so that the relationship between energy usage and weather can be observed across a full range of annual temperature conditions.

Using consumption data over a partial year may lead to poor predictions for weather conditions that were not observed in the baseline period. For example, if the baseline period spans from

October to April, the baseline period model will not have coverage of hot summer weather conditions. Consequently, the model will have to predict out-of-sample to estimate energy usage on a 95-degree day in July. Predicting out-of-sample refers to predictions that are outside of the range of the independent variables used in the regression model.

3.3. Step 3 - Clean the Data

It is vital that the collected baseline data accurately represent the operation of the facility or system before improvements were made. Anomalies in the data can have a large effect on the outcome of the analysis. Thus, after collecting the baseline data (for the dependent variable of interest and any relevant independent variables), one should spend some time reviewing and "cleaning" the data.[14] Data cleaning efforts, which should be conducted on both the baseline period data and the reporting period data, typically include:

- **Examine data outliers**. Identify data points that do not conform to the distribution observed for most of the data and seek an explanation for their unusual values. Atypical events that result in outliers include equipment failure, situations resulting in abnormal facility closures, and malfunctioning of the metering equipment. Truly anomalous data should be documented and then removed from the data set, as they do not describe the facility or system.
- **Make any adjustments related to non-routine events.** Non-routine events include renovations, facility expansion, equipment addition or removal, changes to occupancy type or schedule, and other one-time only or infrequent events. For a discussion of how to make non-routine event adjustments, refer to Section 3.1.7 of the Verification by Energy Modeling Protocol.
- **Identify and address missing data values**. If there are large gaps in the data, seek an explanation and/or alternative data sources, such as a nearby weather station. If data are missing for a relatively small set of observations, they should be filled in. The practitioner can fill in a handful of missing hourly temperature values, for example, via simple averaging (taking the average of the previous reading and the next reading). A more robust approach, appropriate for more than a handful of missing observations, is to interpolate the values via regression modeling (that is, creating a regression model to predict the missing values).
- **De-duplicate the data**. Utility billing and interval data can be susceptible to duplication. For example, you might have two records for the hour ending 10:00 AM. Those two records could be exact duplicates, or they might differ slightly. If possible, determine why the duplication occurred. Regardless, you will want to eliminate records for the same

[14] The timing of data cleaning by engineers and by economists commonly diverges. Economists typically collect and clean both the baseline and the post-installation data as part of Step 2 and conduct the subsequent steps on the entire pre- and post-period. Engineers typically collect, clean and model baseline data and then turn to the post-installation data.

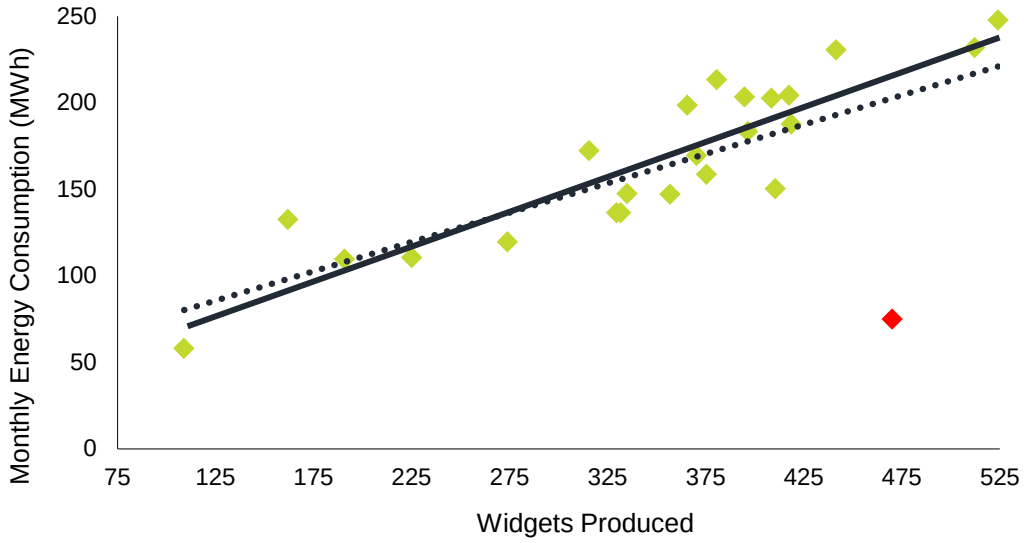
timestamp. If the records are indeed exact duplicates, just drop one of the records. If one record is 0 kW and another has a non-zero value, drop the 0 kW record (assuming an actual read of 0 kW is atypical). If the two records have the same timestamp but different kW values, perhaps take the average.

- **Convert all data sources to a common time zone**. If you intend on using weather data in your analysis, it is critical that the time zone of the weather data matches the time zone of the consumption/demand data (and any other data used in the analysis). Also, be mindful of records that could be affected by daylight savings time.
- **Standardize all measurements to a common time unit**. The observation interval must be consistent across all variables. For example, a regression model using monthly utility bills as the outcome variable requires that all other variables originally collected as hourly, daily, or weekly data be converted into monthly data points. In such a case, it is common practice to average (or, if more appropriate, sum) points of daily data over the course of a month, yielding synchronized monthly data. When working with monthly data, practitioners are encouraged to use daily averages rather than monthly sums, as the number of days in each month varies.
- **Examine scatterplots of the dependent variable versus each independent variable to determine if any regression outliers are present**. Most commonly, one graphs the independent variables on the X axis and the dependent variable on the Y axis. A "regression outlier" is a point in the scatterplot that does not fit the overall trend. Such outliers pull the estimated regression model in their direction, likely leading to a worse overall fit (and worse fit statistics like root mean squared error (RMSE) and R^2, discussed further in Section 5).

  Seek an explanation for the occurrence of any regression outliers and remove them if they are truly anomalous data points, not representative of operating conditions. As an alternative to removing the outliers, the practitioner could employ a regression approach that reduces the impact of outliers, such as one based on the mean absolute error.

  Figure 3-1 shows an example of a scatter plot of monthly MWh and the number of widgets produced per month. Note that there is an overall linear trend, but there is one regression outlier (in red). Also note that this regression outlier is not necessarily an outlier in terms of monthly MWh or number of widgets produced – it only stands out when the two variables are compared. The dotted linear trend line describes the full data set and is pulled in the direction of the outlier. The solid linear trend line describes the relationship without the regression outlier and does a better job of capturing the overall linear trend.

Figure 3-1: Regression Outlier



3.4. Step 4 - Graph the Data

Though subsumed in the previous step, graphing the data warrants a step of its own, as one of the key requirements for using a linear regression model concerns the relationship between the dependent variable and the independent variable(s). For linear regression to be a valid, defensible approach, a scatter plot between the dependent variable and the independent variable(s) must show an approximate linear trend. This requirement is the pillar of linear regression modeling. Practitioners should examine scatter plots between the dependent variable and each of the independent variables used in the model. As an example, Figure 3-1 illustrates a scatter plot for the linear relationship between monthly energy consumption and the number of widgets produced per month (with the dependent variable graphed on the Y axis).

When a scatterplot shows multiple distinct linear trends, it is important to investigate whether those trends correspond with a data category. Common examples include weekday vs. weekend load patterns and occupied vs. unoccupied operating hours. Section 3.2.2 of the *Verification by Energy Modeling Protocol* includes detailed guidance about using categorical variables in regression models.

Indicator variables, commonly used in regression analysis, are binary (0, 1) categorical values that indicate presence or absence of a condition expected to shift the outcome. Note that a scatter plot between the dependent variable and any indicator variables will not show much of the trend since the indicator variable takes only one of two values. Still, the use of indicator variables is encouraged, especially if they make a statistically significant contribution to dependent variable prediction. The concept of *statistical significance* is discussed in slightly more detail in Section 5.2.3.

3.5. Step 5 - Select and Develop Model

After verifying that the relationship between the dependent variable and the independent variable(s) is approximately linear, one can begin developing regression models. To create a baseline equation, perform a regression analysis on the measured variables.

The equation calculated from the regression analysis represents the baseline relationship between the variables of interest. Figure 4-1 in Section 4.2 shows the data and the model estimated for the value of the outcome variable as a function of one independent variable – a *simple regression*.

Frequently, however, more than one independent variable influences the outcome variable. For example, the electricity used by a chiller system might be affected by variations in outside temperature, relative humidity, hours of facility use, and number of occupants. To accurately model cooling energy consumption, we need to include multiple independent variables, creating a *multiple regression* model. Subsequent sections provide more detailed explanations of model development, with examples of multiple regression analysis given in Section 4.4.

3.6. Step 6 - Validate Regression Model

Once you have created as baseline (or pre-post) model, there are statistics you should calculate (or, more appropriately, have software calculate for you) to assess (1) whether or not your independent variables make significant contributions to the prediction of your dependent variable, (2) the goodness-of-fit of your model, and (3) the accuracy of your model. These statistics will be noted here and discussed in greater detail in later sections.

To assess the significance of contributions made by independent variables, some relevant statistics are:

- *F-statistic* – Section 5.1.7
- *t-Statistics* – Section 5.2.2
- *p-values* – Section 5.2.3

To assess model goodness-of-fit, some relevant statistics are:

- R^2 – Section 5.1.1
- *Adjusted* R^2 – Section 5.1.1

To assess model accuracy, some relevant statistics are:

- *Root Mean Squared Error* (RMSE) – Section 5.1.4
- *CV*(RMSE) *Coefficient of Variation of the Root Mean Squared Error* – Section 5.1.5
- *Net Determination Bias* – Section 5.1.6

For multiple regression models, it's also advisable to examine variance inflation factors (VIFs). VIFs can identify whether or not multicollinearity (which occurs when the independent variables

are strongly correlated with each other) is a concern. Multicollinearity and VIFs are discussed in greater detail in Section 5.1.8.

Another popular approach to testing the accuracy of a regression model is called out-of-sample testing. This entails splitting your original data set into a training and a testing data set (the training data set is typically larger). The practitioner uses the training data set to create the regression model and uses the testing data set to test the accuracy of the model (that is, compare predicted values to actual values). Out-of-sample testing is commonly used iteratively via a technique called Monte-Carlo cross-validation. Section 5.3 discusses out-of-sample testing in greater detail.

3.7. Step 7 - Analysis of Residuals

It is rare for a regression model to make predictions that are correct 100% of the time. There is generally a difference between the predicted values and the actual, observed values. This difference is referred to as the residual (where residual = actual value – predicted value). Several of the key assumptions made when fitting an OLS (linear) model concern the distribution of the residuals. Namely, the residuals must meet the following conditions:

1. Residuals must follow an approximate Normal distribution with a mean of zero.
2. Residuals must have a constant variance (referred to as homoscedasticity). That is, residuals should not be larger or smaller as the independent variable(s) increases.
3. Residuals must not be correlated with any of the independent variables or the predicted values of the dependent variable.
4. Residuals must be independent of each other. In other words, the residual at time *t* must not be correlated with the residual at time $t - 1$ (or any other period). This type of correlation is referred to as autocorrelation and/or serial correlation.

It is essential for the practitioner to check whether these conditions are met. If these conditions are violated, then conclusions drawn from the regression model could be incorrect. Performing a residual analysis can also help to identify any regression outliers that might have been overlooked. The discussion on residual analysis is continued in Section 5.4. There, the reader can find several methods for checking the conditions noted above.

4. Models

This chapter describes types of linear regression models that are commonly used for M&V. Spreadsheets and statistical software can create simple and multiple regressions – the models most commonly used in M&V, as discussed below. These tools can also develop second-order or higher polynomial functions, logistic regressions, and other types of models, which can be appropriate in certain circumstances. The M&V practitioner should always graph the data in a scatter chart (Step 4 in the process) to verify the type of curve that best fits the data.

The *ASHRAE Inverse Model Toolkit* (*ASHRAE RP-1050*) is a useful tool for automating the creation of the various model types described below.

4.1. One Parameter Model (Mean Model)

Single parameter (1P), or *mean models*, estimate the mean of the dependent variable and are the simplest models described in this guide. They are not really regression models but are included here for completeness. A mean model would describe energy use that is not related to other independent variables, such as that of a light that runs continuously.

4.2. Two Parameter Model (Simple Regression)

Two parameter (2P) models are the simple linear regression models with which most M&V practitioners are familiar. They are appropriate for modeling building energy use that varies linearly with a single independent variable, such as ambient temperature. In most commercial buildings, metered whole-building energy use varies linearly with ambient temperature above 75º F due to changes in cooling energy use.

A linear least squares regression with only two parameters is often called a *simple* regression. The equation below is the standard form of a simple regression, illustrated in Figure 4-1 with actual building data.

- **Simple Regression:** $Y = \beta_1 + \beta_2 X_1$

  where: Y = the value of the dependent variable

  β_1 = the parameter that defines the *y-intercept* (the value of *y* when *x* equals zero)

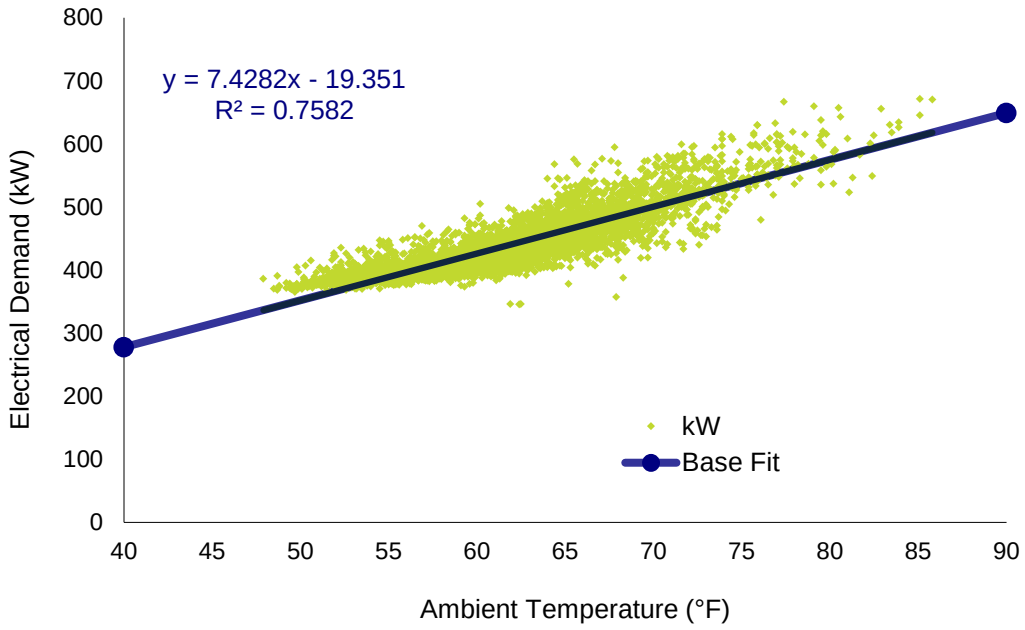
  β_2 = the parameter that describes the linear dependence on the independent variable (*slope*)

  X_1 = the value of the independent variable

(Note that statisticians typically describe this model as $Y = \beta_0 + \beta_1 X_1$. In this text, we use the former notation, as it is consistent with the common engineering terminology *two parameter model*.)

The following graph is an example of a simple regression.

Figure 4-1: Electrical Demand vs. Ambient Temperature



4.3. Simple Regression Change-Point Models

Some systems are dependent on a variable, but only above or below a certain value. For example, cooling energy use may be proportional to ambient temperature, yet only above a certain threshold. When ambient temperature decreases to below the threshold, the cooling energy use does not continue to decrease, because the fan energy remains constant. In commercial buildings with economizer cooling, this threshold is often 55º F. Similar behavior is often seen in building gas usage, because the heating energy is proportional to ambient temperature during the space heating season and the energy associated with hot water use is constant across all seasons.

In cases like these, simple regression can be improved by using a *change-point* linear regression. Change point models often have a better fit than a simple regression, especially when modeling energy usage for a facility. Because of the physical characteristics of buildings, the data points have a natural 2-line angled pattern to them, that is, display a linear relationship that changes (has a different slope) at a given point. Sometimes it is even appropriate to use multiple change points.

The practitioner interested in estimating change-point models should consult BPA's *Verification by Energy Modeling Protocol*, Chapter 3, for a complete discussion of change-point models.

4.4. Multiple Regression

The simple regression and change-point models discussed thus far have all used a single independent variable. Of course, for many building systems, energy use is dependent on more than one variable. In such cases, single variable models will typically result in low R^2 values. When using only one independent variable, the equation has only limited ability to predict the dependent variable, because it does not account for other key factors that should be present in the model.

In such cases, including other variables that are known to influence energy usage will provide a more accurate model. Commonly used variables whose variation is related with variation in energy use include: hours of occupancy in buildings, number of employees on given day, meals served at a restaurant, amount of conditioned floor space, equipment or appliances in use, and water usage. Including two or more independent variables produces a multiple regression model.

Simple regression can be visualized as fitting a line. Multiple regression models with two independent variables fit a plane, and a three-variable model fits a 3-dimensional space. The general format of the model is.

- $Y = \beta_1 + \beta_2 X_1 + \beta_3 X_2 + \beta_4 X_3 + \ldots + \beta_i X_{i-1}$

  where: i = the number of predictors

Note that in common statistics terminology, *multiple regression* typically refers to regression models with two or more independent variables and a single dependent variable. In *multivariate regression*, by contrast, there are multiple dependent variables and any number of predictors. The *ASHRAE Inverse Model Toolkit* refers to multiple regression models and change-point models with multiple independent variables as *multiple-variable* or *multi-variable* models.

Note that additional independent variables will always improve the model's fit (as measured by R^2) regardless of whether or not those variables help in the prediction of observed usage. However, this does not necessarily mean that the model is improved. With multiple regression models, practitioners should refer to adjusted R^2 rather than R^2. The "adjusted" version of this statistic essentially attaches a penalty for each additional explanatory variable in the model. See Section 5.1.2 for more discussion on adjusted R^2.

4.4.1. Categorical Variables

Energy use modeling can account for change of states (broadly, the influence of categorical variables, defined and discussed in this section) by estimating separate models for each state, estimating a single model with categorical variables, and estimating change-point models (a specific form of a model with categorical variables, described in the previous section). Most energy models for M&V will have only one continuous independent variable but may also incorporate categorical variables.

Variables can be divided into two general types: *continuous* and *categorical*. Continuous variables are numeric and can have any value within the range encountered in the data. Continuous variables are either interval or ratio numbers (where a value of 10 is twice the magnitude as a value of 5). Continuous variables are measured things, such as energy use or ambient temperature. Categorical variables include things like daytype (weekday or weekend, or day of week), occupancy (occupied or unoccupied), and equipment status (on or off). As examples, *occupancy status* is a categorical variable, while *number of occupants* is a continuous variable.

For use in a regression analysis, any categorical variable must be expressed in a binary form, such as taking the value of 1 for Monday and taking the value of 0 for all other days. This is because all the variables in a regression model must be linearly related to the dependent variable. A conceptual category such as day-of-week therefore cannot be included in a regression if it takes values such 1 for Monday, 2 for Tuesday, on up through 7 for Sunday; Tuesday does not have twice the impact on the dependent variable than Monday, nor does Wednesday have three times the impact.

As mentioned at the end of the prior section, one needs to take care in adding additional variables – such as multiple binary variables to describe a composite concept (such as day-of-week) – because the model can become over-specified, and the parameter estimates inaccurate and imprecise. Thus, when needing to create a set of binary variables to capture a composite categorical concept, the M&V practitioner should consider the most concise way to express the underlying relationships between these categories and the dependent variable. Continuing with the day of week example, it may be that activity ramps up during the week; appropriate categories might be Monday/Tuesday, Wednesday/Thursday/Friday, and Saturday/Sunday, where *Mon_Tues* has the value of 1 if the day is a Monday or Tuesday and 0 otherwise, and similarly for the other variables.

Finally, when working with binary variables describing composite categories, the modeler includes one less binary variable in the equation than the total number of categories in the set. Continuing with the example, when the variables *Mon_Tues* and *Wed_Thus_Fri* both have the value of 0, the day must be a Saturday or Sunday; it would be redundant (that is collinear) to add the variable *Sat_Sun*.

According to *ASHRAE RP-1050*, practitioners using categorical variables commonly err by inappropriately using them only to change the line's intercept. The M&V practitioner needs to carefully consider whether the categorical variable is expected to affect the model's intercept term, a slope term, or both. If the slope likely differs among categories, the model must include terms to capture the interaction of the categorical and continuous variable, which can be tedious and error-prone to accomplish in Microsoft Excel. (Another solution is to fit separate models for different levels of the categorical variable.)

An appropriate statistical approach to apply with categorical variables is the *General Linear Model* (GLM). Multiple regression is typically used where the independent variables are continuous, but a general linear model can accommodate both categorical and continuous predictor variables. In avoiding the common pitfall of all categories having the same slope, it is important to use the proper GLM method. (Please refer to a statistics text for further discussion

of general linear models. Some resources are noted in the References and Resources section of this document.)

Instead of using a multiple regression of the format in *ASHRAE RP-1050,* you can create separate models for each category or combination of categories, and then combine these individual models into a complete model. The basic process is similar to using *IF* statements to determine, for each data point, the category of the categorical independent variable, and then using the intercept and slope that are appropriate for that category.

4.5. Uncertainty and Confidence Intervals

4.5.1. Uncertainty

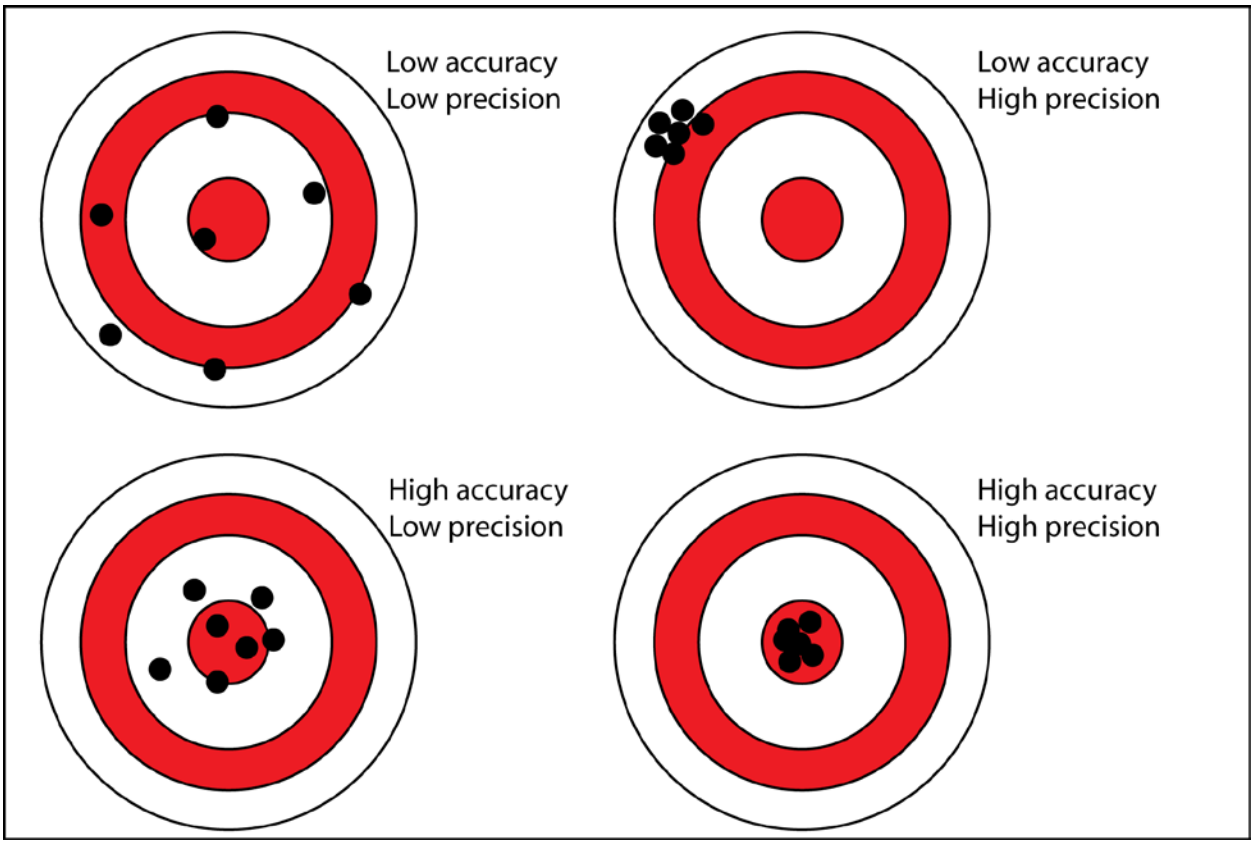
Regression analysis yields estimates, predictions that will not be 100% accurate. Thus, modelers speak of the uncertainty of the estimates, that is, uncertainty in the predicted *y-value*. Uncertainty in regression analysis results from three principal sources:

- Measurement uncertainty or measurement error,
- Coverage error, and
- Regression uncertainty or model uncertainty.

Measurement Uncertainty

Measurement uncertainty has two principal components: measurement bias and measurement precision. Bias relates to issues of calibration and accuracy; precision relates to the magnitude of random variation that occurs when multiple measurements are made. Figure 4-2 illustrates these concepts. The concept of measurement uncertainty as it relates to regression analysis pertains to the independent variables, as any measurement error in the dependent variable contributes to model uncertainty, with the error contributing to the model residual.

Figure 4-2: Accuracy vs. Precision



Instruments for acquiring measurements should be of sufficient resolution and precision that the uncertainties in measurements are small relative to the regression uncertainty. Measurement bias due to measuring equipment error should be eliminated through calibration, and careful instrumentation design and installation should be used to minimize other measurement bias errors. Installation criteria for accurate measurement, such as the need for a straight duct of a specific number of equivalent duct diameters for a flow measurement, may be important.

Note that, even though an installation limitation may introduce the same bias to the pre and post periods, the fact that the bias is the same may not mean that the savings estimate is not biased. Whether or not there is a savings bias is dependent upon the type of bias (that is, additive or multiplicative) and how the measurement is mathematically used.

As applicable and possible, utility meters should be used for energy-use measurements. By M&V convention, utility meter data is considered to have zero uncertainty for savings estimates. Similarly, data from a nearby National Oceanic and Atmospheric Administration (NOAA) weather station should be used for weather measurements, but such measurements should be verified to be representative of the conditions at the treated building. NOAA sites are far less likely to have biases or inaccuracies due to solar effects and sensor calibration errors than site measurements.

Specific to weather data, check for evidence of instrumentation changes over time. For example, one might take differences with nearby weather station data and plot over time. Some stations

may also document such changes. A change in a weather data measurement source during the baseline or post period may require: (1) the inclusion of an indicator variable for the effected period in the regression model, (2) a normalization of the weather data, or (3) a full update to the original energy model.

For a thorough discussion of measurements, refer to *Section 6, Instrumentation*, and *Annex A, Physical Measurements*, within *ASHRAE Guideline 14, Measurement of Energy, Demand, and Water Savings*.

Coverage Error

Coverage error occurs when an M&V data set does not fully "cover" the range of conditions that drive energy use, which is the full range of building or system operating conditions. As stated in Section 3.2, measurements should be conducted for a sufficient period to capture a significant range of the independent variable(s). Beyond that, no definitive criteria can be provided regarding the sufficiency of shorter-term data for annual extrapolation. *ASHRAE Research Project 1404, Measurement, Modeling, Analysis and Reporting Protocols for Short-term M&V of Whole Building Energy Performance* provides some guidance.

In a production environment, the consistency of production will determine this length of time. When weather is the independent variable, the season and climate will determine the length of time necessary. If seasonal variations in weather are minor, a relatively short time may be possible and still cover a wide range of conditions. If seasonal variations are significant, longer periods (up to a year) may be advisable.

Measurements of the dependent and independent variables must cover the same time periods.

Regression Uncertainty

Regression uncertainty (also referred to as savings uncertainty) results both from modeling errors – explanatory variables are omitted from the model or an incorrect functional form is specified – and because people's unpredictable behaviors affect energy use. Uncertainty in regression typically refers to the uncertainty in the output from a regression; uncertainty in the regression coefficients is typically referred to in a more explicit manner as the *uncertainty of the slope*.

A goal of any M&V plan should be to minimize uncertainty in the savings estimate (regression uncertainty). More specifically, the goal should be to make the uncertainty small relative to the savings. *ASHRAE Guideline 14-2014, Annex B* refers to this as the *fractional savings uncertainty (FSU)*.[15]

Generally, factors that affect regression modeling uncertainty include:

- Number of points used in the baseline regression
- Number of points in the post-installation period

[15] Refer to *ASHRAE Guideline 14-2014, Annex B: Determination of Savings Uncertainty* for a more detailed discussion of savings uncertainty than is provided here.

- Number of significant independent variables included in the regression

One way to reduce the fractional savings uncertainty is to use more data. Gathering data over a longer period, and/or at more frequent intervals, will generally reduce the uncertainty. Note, though, that as data is gathered at more frequent intervals, this will increase serial autocorrelation – each reading becomes more significantly related to the prior reading. Uncertainty estimates must account for this autocorrelation. Costs may be affected by increasing the length of time required to collect data or monitoring additional variables.

Another way to reduce the fractional savings uncertainty is to include more relevant independent variables. The *t-statistic* and *p-value* should be used to check for the relevance of additional independent variables.

As with all M&V protocols, the emphasis on accuracy needs to be balanced against the level of savings and cost. Factors affecting regression uncertainty should be assessed to determine the amount of effort and cost needed to increase accuracy.

4.5.2. Confidence Level and Confidence Interval

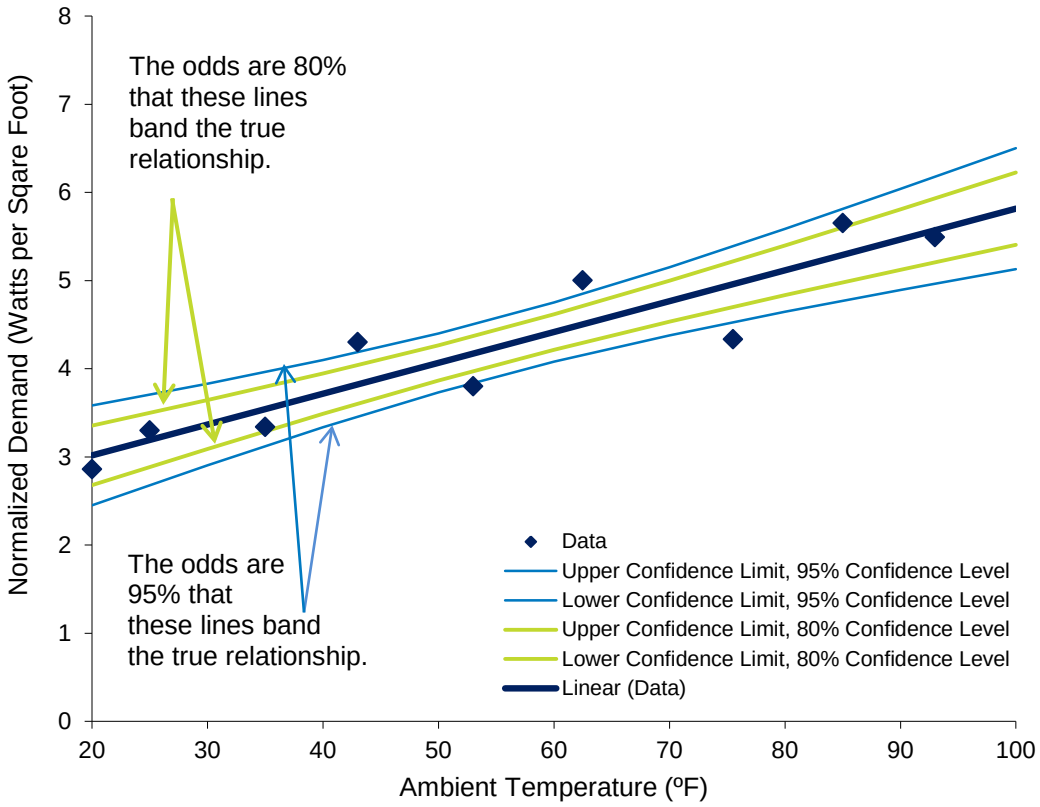
Uncertainty is associated with a given confidence level or probability – for example, "We are 90% confident that the range 433 and 511 kWh bands the true value," or, as it is more commonly but less accurately expressed, "We are 90% confident that the true value lies between 433 and 511 kWh." Confidence level is an input number; for a given sample and regression, the higher the confidence level specified, the larger the estimated range that is likely to contain the true value that proportion of the time.

Confidence intervals are a common way to express uncertainty. A 95% confidence level implies that there is a 95% chance that the *confidence interval* resulting from a sample contains the true value. Confidence intervals define the range – an uncertainty band – that is expected to band the true relationship between the dependent and independent variables, with a certain probability. The width of the confidence interval provides some idea of uncertainty about the estimated values. For example, the results of a regression analysis of savings may be reported as "500 kWh ±5% at the 95% confidence level." This means that there is a 95% chance that the confidence interval of 475 to 525 kWh contains the true value of savings. A statement of "500 kWh ±5% at the 68% confidence level" means that there is only a 68% chance that the true savings value is between these limits, and a 32% chance that it is outside them.

The practitioner should note that the true value does not fluctuate; rather, because of regression uncertainty (and, perhaps, measurement uncertainty), there cannot be complete certainty that the true savings value lies within these limits. Confidence limits are the bounds of the confidence interval.

Figure 4-3 provides a graphical representation of confidence intervals. The bounded confidence intervals in this figure demonstrate that higher chances an interval contains the true regression line require wider intervals than lower chances (that is, the wider the confidence interval, the more likely it is to contain the true value). The lines in this figure represent upper and lower confidence limits.

Figure 4-3: Confidence Intervals for a Regression

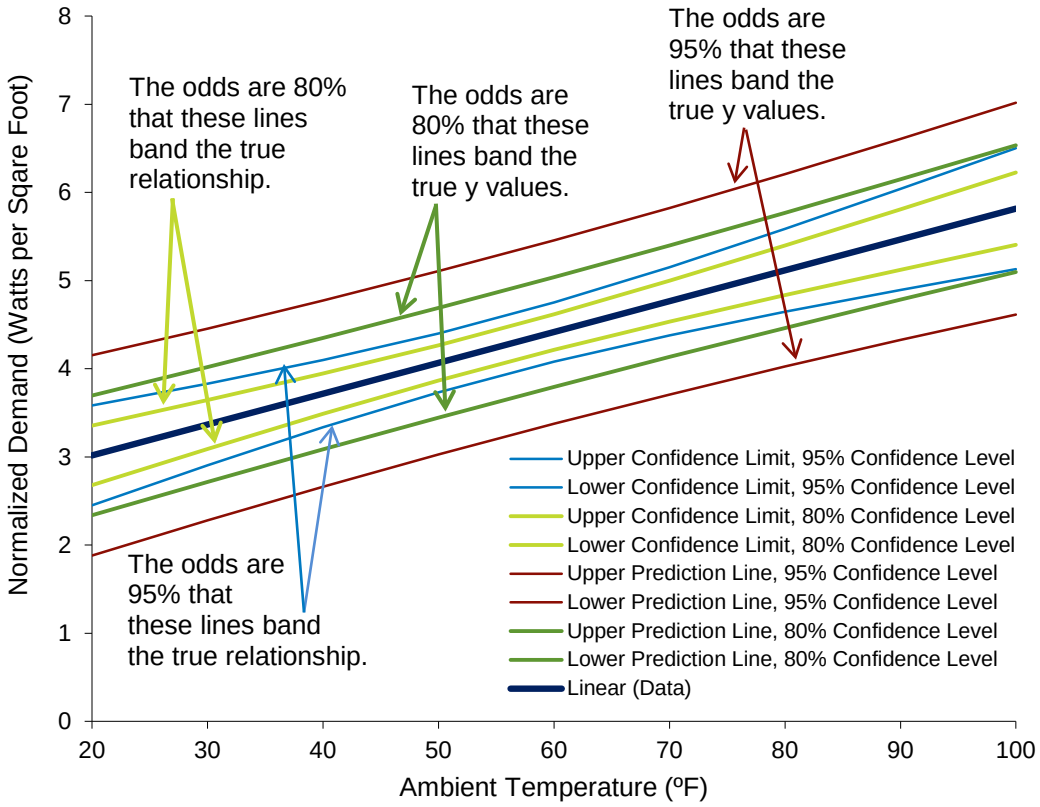


4.5.3. Prediction Interval

Prediction intervals are like confidence intervals, but rather than estimating the distribution of a true value (such as *average* demand), prediction intervals provide a range of values within which a single value is expected to fall. As an example of the distinction, a confidence interval can provide a range of values within which *average* demand is expected to fall when the ambient temperature is 60ºF. A prediction interval can provide a range of values within which observed demand is expected to fall when the ambient temperature is 60ºF. Prediction intervals are wider than confidence intervals since, under the identical conditions, it is more difficult to predict the value of a future point than it is to predict the distribution of the population mean.

Figure 4-4 illustrates prediction intervals, adding them to Figure 4-3, above.

Figure 4-4: Prediction Intervals for a Regression



4.5.4. Confidence Levels and Savings Estimates

Savings estimated from regression analyses should describe the range of values corresponding to a given confidence level. If a single savings estimate, rather than a range, is required, the savings estimate should be the mean point estimate (that is, the value that falls directly in the middle of the confidence interval).

The less scatter, or variability, in the data, the narrower the confidence intervals; greater scatter results in winder confidence intervals. However, regardless of the degree of scatter, the confidence interval will be wider when requiring a higher probability that it contains the true regression line or the true value of savings than when requiring a lower probability. For example, the interval estimated for a 99% confidence interval will be wider than it will be for a 95% confidence interval.

For a single value of savings, requiring a greater probability that an interval contains the true value results in a wider uncertainty band, which in turn results in a lower estimate of minimum likely savings. If a lower probability is acceptable, the uncertainty band will be narrower and the estimated minimum savings will be higher. To summarize, the minimum savings estimated is higher with a lower confidence level and is lower with a higher confidence level.

5. Validating Models

5.1. Statistical Tests and Measures for the Model

After developing the regression model, you must assess its *goodness of fit*. There are many ways of testing regression models. The following is an engineering layperson's description of some of the statistical measures and methods used as guidance for validating models. Interim measures needed for the statistical tests, such as *root mean squared error*, are also described in this section.

5.1.1. R-Squared (Coefficient of Determination)

The *coefficient of determination* (R^2) provides a measure of how well the independent variables explain variation in the dependent variable. R^2 values range from 0 (indicating none of the variation in the dependent variable is associated with variation in any of the independent variables) to 1 (indicating all of the variation in the dependent variable is associated with variation in the independent variables, a "perfect fit" of the regression line to the data). The rule-of-thumb for an acceptable model using monthly billing data is an $R^2 > 0.75$.

If the R^2 is low, you may wish to return to Step 5 in the regressions process (see Chapter 3) and select additional independent variables that may explain energy use and add them to your model; then use the adjusted R^2 (see Section 5.1.2) as a goodness-of-fit test for a multiple regression.

The R^2 value can be thought of as a goodness-of-fit test; but a high R^2 value is not enough to say the selected model fits the data well, nor that a low R^2 indicates a poor model. Professional judgment should be applied, and other fit criteria in addition to R^2 should be assessed. For CV(RMSE) (see Section 5.1.5), a low value (often interpreted as 10% or 15%) is desirable. For example, a model with a low R^2 is acceptable when there is a clear relationship between the dependent and independent variables, as evidenced by the following: The scatter of the observed *y*-values around the regression line is low, yet large in relationship to the total scatter of *y*-values from the mean of *y*, and total *y* scatter is much smaller than the total scatter of *x*-values from its mean (this results in a low slope estimate). In a situation where the total scatter of *y* and *x* compared to their means is more comparable, a low R^2 can be acceptable when the estimated coefficient of *x* is significant, despite the unexplained variation; however, there will be relatively high uncertainty in the resulting savings estimates.

The calculations for estimating uncertainty are described in Section 4.5.

5.1.2. Adjusted R-Squared

In multiple regression models, the addition of an independent variable will always result in an increase in the model's R^2, which means the basic R^2 value is not an appropriate indicator of model fit. Instead, one should judge model fit using adjusted R^2, a value produced by adjusting R^2, dividing R^2 by the associated degrees of freedom (discussed next). The value of the adjusted

R^2 only increases from one model specification to another if the additional independent variable(s) improve the model more than by random chance.

5.1.3. Degrees of Freedom

Degrees of freedom (DF) is a common input for statistical calculations. Degrees of freedom is the number of values in a calculation that are free to vary and is calculated by subtracting the number of parameters in the model from the total number of data points.

5.1.4. Root Mean Squared Error

Root mean squared error (RMSE) is an indicator of the scatter, or random variability, in the data, and hence is an average of how much an actual *y*-value differs from the predicted *y-value*. It is the standard deviation of errors of prediction about the regression line.

5.1.5. Coefficient of Variation of the Root Mean Squared Error

Coefficient of variation of the root mean squared error – CV(RMSE) – is the RMSE normalized by the average *y-value*. Normalizing the RMSE makes this a nondimensional that describes how well the model fits the data. It is not affected by the degree of dependence between the independent and dependent variables, making it more informative than R-squared for situations where the dependence is relatively low.

5.1.6. Bias

Bias refers to any systematic differences between actual energy use and that predicted by a regression model. It can result from many parts of the analysis process, including mis-specified regression models or a lack of coverage in the independent or dependent variables, among others. *Energy* models should always be checked for bias: Does the model accurately re-create the actual baseline energy use on average? *Demand* models, on the other hand, generally do not require a bias check, since demand is not summed over time. Also, demand models will generally not require different points to have different weights, so that potential for bias error (from not using a weighted regression when one is warranted) is not a concern. Since regression itself minimizes the error for each point, there will typically be no need to check bias for a demand model. M&V practitioners should take care to understand any unique situations that may require checking for bias in a demand model.

Two indices are defined in *ASHRAE Guideline 14* for checking energy model bias. These two indices are *net determination bias error* (or *mean bias error*) and *normalized mean bias error*. Be forewarned that the Guideline is somewhat confusing, since these two indices are nearly the same and the document refers to one of the indices using two different terms.

Net determination bias is simply the percentage error in the energy use predicted by the model compared to the actual energy use. The sum of the differences between actual and predicted energy use should be zero. If the net determination bias = 0, then there is no obvious bias.

ASHRAE Guideline 14-2014 accepts an energy model if the net determination bias error is less than 0.005%.

Often, bias may be minor, but it still will affect savings estimates. If the savings are relatively large compared to the bias, bias may not be important. But in many cases, bias could be influential.

- **Net Determination Bias Error (NDBE)**: $NDBE = 100 * \frac{\sum_i (E_i - \hat{E}_i)}{\sum_i E_i}$

- **Normalized Mean Bias Error (NMBE):** $NMBE = 100 * \frac{\sum_i (E_i - \hat{E}_i)}{(n - p) * \bar{E}}$

In the equations above, E_i represents actual energy usage, $\hat{E}_i$ represents predicted energy usage, $\bar{E}$, represents average energy usage, n represents the dumber of data points, and p represents the number of explanatory variables in the model. Note that the two indices are identical if, in NMBE, p = 0. Therefore, the only difference between the two bias error calculations is an adjustment for the number of parameters in the model.

Since there is no averaging occurring, it seems that *mean bias error* is a misnomer. The *net determination bias error* is simply the percentage error in total energy use predicted by the model over the relevant (baseline) time period. In the equation for *normalized mean bias error*, there is an average term in the denominator, but the result is still simply a percent error, which is adjusted for the number of parameters in the model.

Regression models *by themselves* will not typically have any bias if created properly. However, as stated above, there can be bias when using regression models, either because multiple categories need to be considered, or because an unweighted regression was used when data points should not have equal weights.

Checking for model bias is an important part of model validation, but there is little value in using *both* of these very similar bias calculations. Keep it simple and just use *net determination bias error*, which provides a net percentage error in the model.

To clarify some of the confusion between guidelines, we have listed the terms and uses for various guidelines below.

- **Normalized Mean Bias Error** – is called *net mean bias error* in the *Guidelines for Verifying Existing Building Commissioning Project Savings*.
- **Net Determination Bias Error –** is called by this same term in the *Guidelines for Verifying Existing Building Commissioning Project Savings*.
- **Mean Bias Error –** is referenced by *ASHRAE Guideline 14* in *6.3.3.4.2.2 Statistical Comparison Techniques*, but the verbal definition of this term is the same as the equation for *net determination bias error*.

➡ **Net Determination Bias** – is a term not found in the statistical literature. References on the Internet point exclusively to *ASHRAE Guideline 14*. Consider *net determination bias* as simply a percentage error.

5.1.7. F-Statistic

The *F-statistic* is similar to the *t-statistic* (described subsequently) but is for the entire model rather than for individual variables. When testing a model, the larger the value of *F*, the better.

In the Excel Regression tool output, *Significance F* is the whole-model equivalent of *p*-value for an individual variable. For a simple regression (no change points) with a single independent variable, the *Significance F* value is the same as the *p*-value for the independent variable. It is the probability that the model does *not* explain most of the variation in the dependent variable. Therefore, low values for *Excel's Significance F* are desirable.

5.1.8. VIFs and Multicollinearity

With multiple regression, models should be checked to avoid multicollinearity. *Multicollinearity* describes a strong relationship between two or more of the *independent* variables. Broad discussion of multicollinearity is beyond the scope of this document. The key point is that allowing multicollinearity in a model can create a number of problems and lead to incorrect inferences from the model.

Multicollinearity between two independent variables means that standard errors for coefficients are over-emphasized, and therefore larger. The coefficient estimates may change erratically in response to minor changes in the model or the data. Even the signs of coefficients can be incorrect!

Multicollinearity may not reduce the predictive power or reliability of the model as a whole; it only affects calculations regarding individual predictors. Significant relationships between independent variables make it difficult to determine which of the correlated independent variables are most significant – that is, which ones most explain variations in the dependent variable.

As a first step to explore the degree of multicollinearity, practitioners should create a correlation table between all potential independent variables. If any correlations are exceptionally large ($r > 0.75$), then multicollinearity could jeopardize any model that uses those pairs of data. Visualizing the relationship between independent variables via scatter plots can also help to determine if multicollinearity could be an issue.

One measure commonly used to determine whether multicollinearity is present in a regression model is called a variance inflation factor (VIF; the equation follows). For a variable of interest, it's the ratio of total variability to variability that is not explained by the other variables. After creating the regression model, the practitioner should consider calculating VIFs for each independent variable in the model. As a rule of thumb, if any VIFs are greater than 10 (meaning that less than 90% of the total variability is explained by the other variables), then

multicollinearity is a concern. In that case, the practitioner should re-estimate the model without the highest-VIF independent variable.

Note that practitioners should not attempt to use VIFs to assess multicollinearity when the model includes interaction terms. As a workaround, practitioners may fit the model without the interaction terms and calculate VIFs. If all VIFs are less than 10, then the practitioner can re-estimate the model with the interaction terms and assume the resulting model will not be compromised by multicollinearity.

Calculating VIFs may prove tedious, but it is not an exceptionally difficult calculation. For simplicity, let us assume that your dependent variable is A and you intend to use three independent variables: B, C, and D. To calculate the VIF for independent variable B, create a regression model where B is the *dependent* variable, and C and D are the independent variables. The VIF for B is a function of the R^2 value for this model:

$$VIF = \frac{1}{(1 - R^2)}$$

Add-ins in Excel that can streamline this process are available. Common statistical analysis programs like Stata and R have VIF packages built-in.

5.2. Statistical Tests and Measures for the Model's Coefficients

5.2.1. Standard Error of the Coefficient (Intercept or Slope)

The *standard error of the coefficient* is like the *RMSE*, but it is calculated for a single coefficient rather than the complete model. The standard error is an estimate of the standard deviation of the coefficient. (In other words, the standard error of a regression coefficient helps answer this question: How far does the estimated coefficient fall from the true coefficient?) For simple linear regression, it is calculated separately for the slope and intercept: there is a *standard error of the intercept* and *standard error of the slope*. These are necessary to get the *t*-statistic for each.

5.2.2. *t*-Statistic

The *t-statistic* is the coefficient (β_i) divided by its standard error. Within regression, the *t*-statistic is a measure of the significance for each coefficient (and, therefore, of each independent variable) in the model. The larger the *t*-statistic, the more significant the coefficient is for estimating the dependent variable. The coefficient's *t*-statistic is compared with the critical *t*-statistic associated with the required confidence level and degrees of freedom. For a 95% confidence level and many degrees of freedom (associated with a lot of data), the comparison *t*-statistic is 1.96. For smaller data sets, the critical t value can be computed via the T.INV.2T function in Excel (see Table 5-4). Measure the *t*-statistic for every independent variable used, and if the *t*-statistic is lower than the critical value (such as 1.96) for any variable, consider removing that variable from your regression model. Go back to Step 5 (Section 3.5) and consider

if a different model specification is more appropriate. Practitioners rarely review the t-statistic of the model intercept term when assessing the goodness-of-fit.

5.2.3. *p*-value

Loosely, the *p-value* is the probability that a coefficient or dependent variable is not related to the independent variable. Small *p*-values, then, indicate that the independent variable or coefficient is a significant (important) predictor of the dependent variable in your model. When the *p*-value for an independent variable is below a certain significance level threshold (commonly 0.01, 0.05, or 0.10), that independent variable is said to be statistically significant (that is, it makes a significant contribution to the estimation of the dependent variable).

5.3. Out-of-Sample Testing

One common approach to assessing the accuracy of a regression model is called out-of-sample testing. The idea behind out-of-sample testing is to see how accurate your model's predictions are when it encounters new data points. This approach proceeds as follows:

1. Divide your full data set into two data sets. Put most of the records in a "training" data set and place the remainder in a "testing" data set. It is best to randomly assign observations from the full data set to the training and testing data sets. As an example, you might number the records in your data set from 1 to 365 (assuming you have daily data covering a one-year period). Then, using a calculator, Excel, or an online random number generator, randomly select a handful of numbers between 1 and 365. The records corresponding to the randomly selected numbers will become the testing data set.
2. Using the training data set, fit your regression model. That is, the estimated coefficients will be based entirely on the training data set.
3. Use the regression model and values of the independent variables for the testing data set to predict the dependent variable for the observations in the testing data set. Are the predicted values close to the actual values? Ideally, the errors will be relatively small and centered around zero. If the errors are, on average, significantly greater or less than zero, then the regression model does not make very good predictions.[16] Since the goal of a regression model is to make accurate predictions in the reporting period, this is problematic.

Of course, it's possible that the values that were randomly selected to be in the testing data set do not represent the full data set very well simply by random chance. This is where Monte-Carlo

[16] To determine if the errors differ from 0, on average, practitioners may consider using the errors to construct a confidence interval. If 0 is not in the resulting interval, then the model tends to over or under predict. Note that a critical t-value (rather than a critical z-value) should be used when making the interval and that the size of the interval will be driven by the number of data points in the testing data.

cross-validation, which is an iterative process that has grown in popularity with the prowess of modern computing technology, comes into play. This technique is described via example below.

For the example, assume that you have one full year of daily consumption (kWh) and one full year of temperature data. Further, assume the relationship between consumption and temperature is linear. The size of the testing data set and other specific numbers in the example are for expository purposes only and are not intended as guidelines.

1. Without replacement, randomly select 30 observations from your full data set and place them into a testing data set.
2. Fit the regression model using the remaining 335 observations.
3. Plug the temperature values from the testing data set into the regression model and compare the actual consumption values to the predicted values. For each of the 30 records, calculate the error (where error = actual – predicted). Take the average error across all 30 records.
4. Repeat steps 1-3 several times. Each time these steps are repeated, a new average error (final piece of step 3) is calculated. Use descriptive statistics and histograms to summarize these average errors. Ideally, a histogram will show an approximate bell-shape centered near zero. A wider, flatter bell-shaped distribution is indicative of relatively larger prediction errors. A taller, thinner bell-shaped distribution is indicative of relatively smaller prediction errors. (Note that the spread in the histogram will also be a function of the size of the testing data set, with a larger data set having a narrower spread.)

In practice, statisticians might repeat this process 5,000 times or more. The number of iterations is typically a function of computing power, though there may be some diminishing returns; 100 iterations will provide more informative results than 10 iterations, but 10,000 iterations may provide results that are like the results provided with 5,000 iterations. The obvious downside to this approach is that implementing it in Excel may prove quite burdensome. It is easier to conduct Monte-Carlo cross-validation using a common statistical analysis program such as Stata, R, and SAS, but using those programs require some programming expertise (and, in the case of Stata and SAS, a financial investment).

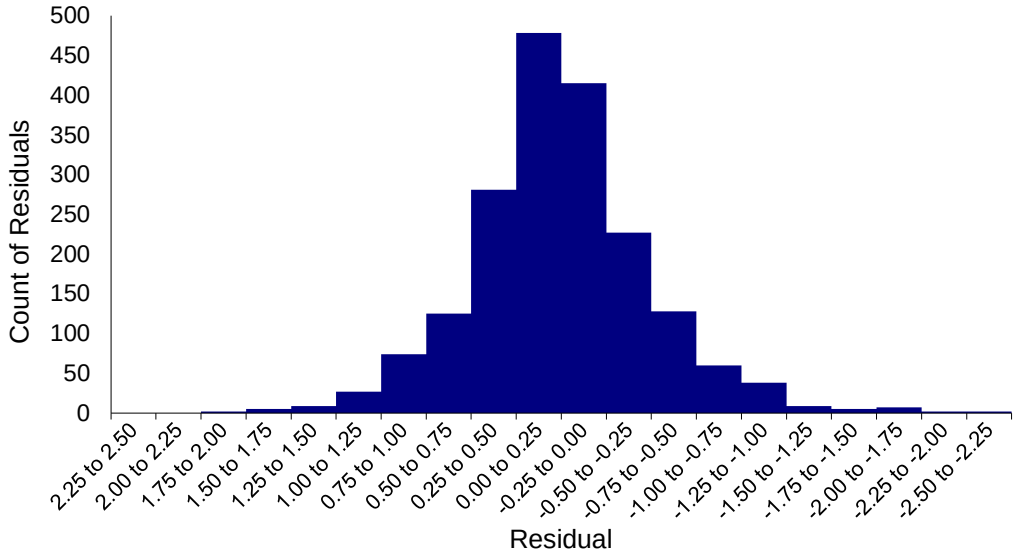
5.4. Analysis of Residuals

As introduced in Section 3, many of the important assumptions that need to be checked when using ordinary least squares regression concern the residuals of the regression model. Violations of these assumptions could lead to incorrect conclusions or overstated significance. This section discusses methods for checking the residual assumptions. Although checking the validity of these assumptions is of essential, residual plots may also reveal other issues. For example, if any curvilinear trends show up, then the regression model may need a higher order term.

5.4.1. Approximate Normal Distribution

A histogram of the residuals should reveal a symmetric, bell-shaped distribution (that is, Normally distributed) centered at zero. Note that real data will never show perfect symmetry around zero, but approximate symmetry is good enough. It is gross departures from normality that are concerning. Figure 5-1 shows an example of an approximately Normal distribution. (Note that model validity necessitates that the residuals are Normally distributed, but the data used to develop the model need not follow a Normal distribution.)

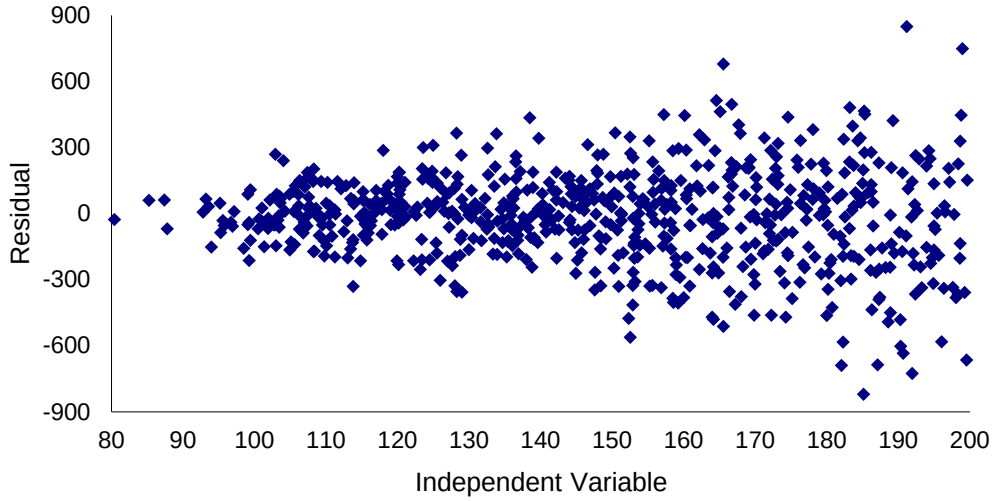
Figure 5-1: Histogram of Residuals



5.4.2. Constant Variance

In addition to being approximately Normally distributed with a mean of zero, residuals must have a constant variance. In other words, the spread of the residuals should not be larger at one end of the range of independent variable values than it is at the other. To check this condition, practitioners can create a scatter plot with the residuals plotted on the Y axis and the independent variable(s) plotted on the X axis. These plots should show random scatter around zero rather than any fanning in/out patterns. Figure 5-2 shows an example where this condition is violated – the spread in the residuals increases for larger values of the independent variable. (Note that these plots can also be used to check that the residuals are not correlated with the independent variables, discussed next.)

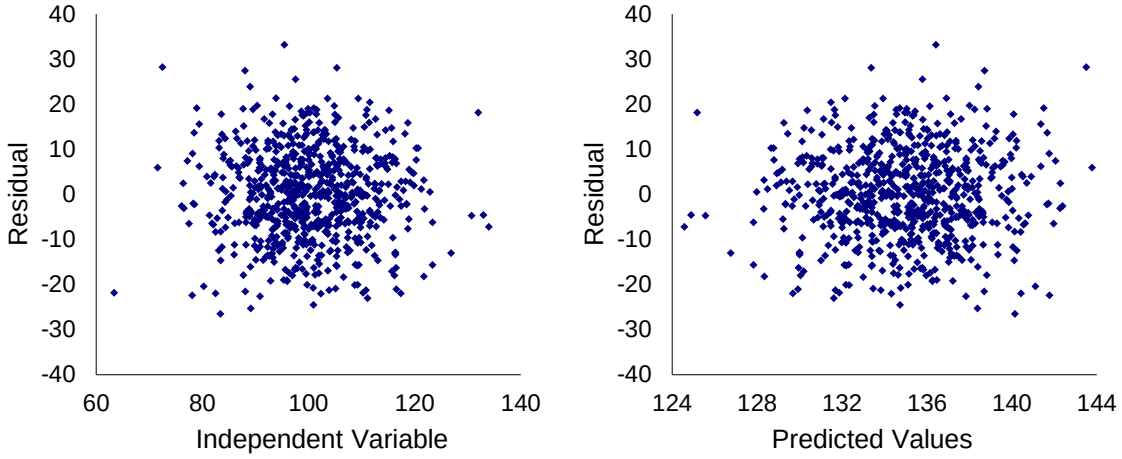
Figure 5-2: Non-Constant Variance in Residuals



5.4.3. Uncorrelated with Independent Variables

The residuals should not show a strong correlation with any of the independent variables in the regression model. To test the validity of this assumption, practitioners can use the same plots examined in the previous section. The scatter plot of the residuals against the independent variable(s) should show no linear patterns. The points should be randomly scattered around zero. This relationship should also hold between the residuals and the predicted values of the dependent variable. The left pane in Figure 5-3 shows residuals plotted against the independent variable, and the right pane shows residuals plotted against the predicted values of the dependent variable. In both cases, there are no trends – just random scatter around zero. (Note that practitioners may see a linear trend when plotting residuals against *actual* values of the dependent variable, but this is not worrisome.)

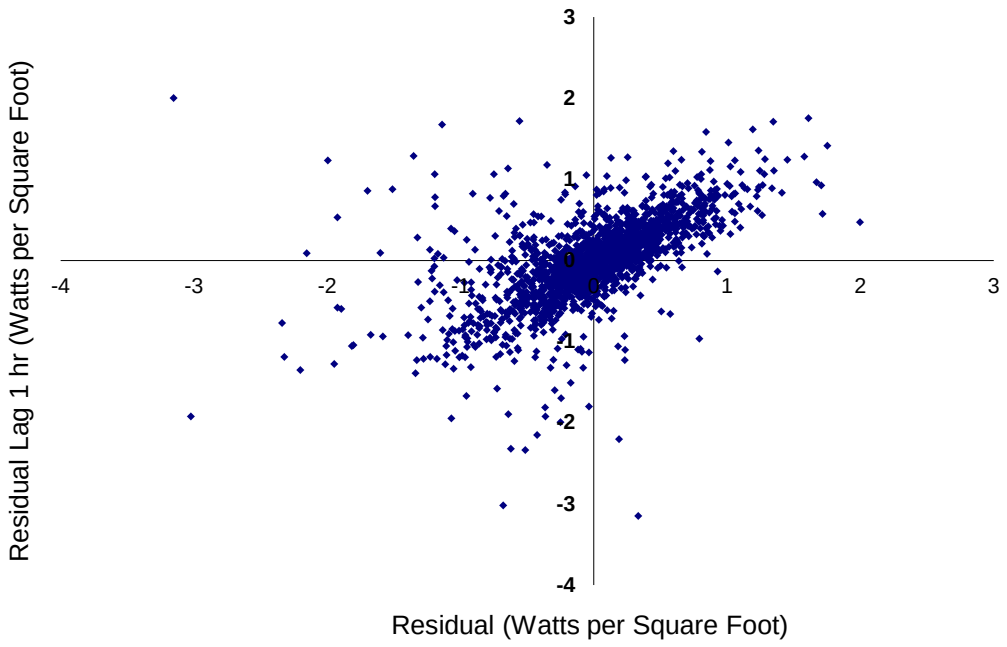
Figure 5-3: Residual Plots



5.4.4. Independently Distributed

The residuals from a regression model are said to be independently distributed if the residual at time t is uncorrelated with the residual at time $t-1$, $t-2$, or any other period. If residuals are found to be correlated with one another, they are said to suffer from autocorrelation or serial correlation. Autocorrelation can be common in energy models, especially with data taken at short intervals. For example, Figure 5-4 is a lag plot of residuals from a model using hourly data. It charts the residuals (X axis) against the residuals from the prior hour (Y axis). Ideally, this plot would reveal no trends. However, that is not the case – the strong relationship shown indicates that this model suffers from autocorrelation. For this model, the autocorrelation should be accounted for; if not, the uncertainty in the model will be underestimated.

Figure 5-4: Residual Lag Plot



The impact of autocorrelation is that the effective number of data points is fewer than the actual number, since the information in each observation is not completely new. A consequence of this is that the variability looks lower than it actually is, making some predictors look significant when they are not. In the equations for the statistical tests, the effective number of data points needs to be substituted for n, the actual number of data points.

Practitioners can estimate the effective number of data points (equation shown below) by first calculating the first-order autocorrelation coefficient, ρ ("rho"). This value is simply the correlation between the residuals and the residuals for the prior time period. The effective number of data points is then given by:

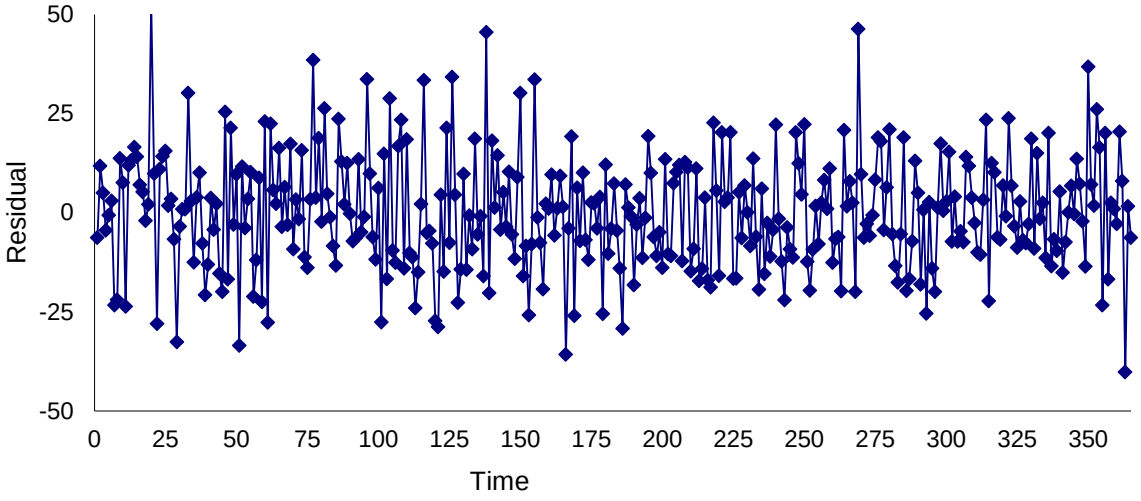
$$Adjusted\ n = (Actual\ n) * \frac{(1-\rho)}{(1+\rho)}$$

Annex D of ASHRAE Guideline 14 suggests that autocorrelation can be ignored for values of ρ less than 0.5. Practitioners can also find a more dedicated discussion on autocorrelation (including potential remedies) in the IPMVP Uncertainty Guide.

5.4.5. Other Plots

When analyzing residuals, there are a variety of other plots that practitioners may consider. A residual time series plot, for example, plots the residuals against time. Such a plot may help identify (1) regression outliers, (2) non-constant variance in the residuals, or (3) autocorrelation. Figure 5-5 shows a residual time series plot for a model that uses daily data. Residuals are plotted on the Y axis and time (which is the date in this case) is plotted on the X axis. This figure is example of an ideal residual time series plot – the spread is constant regardless of the time period, residuals tend to hover around zero for the full time period (rather than hover above zero for a period of time, then hovering below zero for the remainder of the time), and there are no alarming outliers. (Note that a few spikes should be expected just by random chance.)

Figure 5-5: Residual Time Series Plot



5.5. Tables of Statistical Measures

Table 5-1 through Table 5-4, below, present the definitions of the relevant statistical measures, their equation formulas, and their calculation in *Microsoft Excel*.

Table 5-1: Definitions of Regression Model Statistics

Regression Model Statistic	Equation or Definition
n	Number of points
p	Number of parameters
df	Degrees of freedom, = *n* - *p*
y_i	Actual y values
$\hat{y}_i$	Predicted y values
Y_{avg} (or $\bar{y}$)	$= \left(\sum_{i=1}^{n} y_i\right)/n$
X_{avg} (or $\bar{x}$)	$= \left(\sum_{i=1}^{n} x_i\right)/n$
SSQ_{total}	$= \sum_{i=1}^{n} ((y_i - \bar{y})^2)$
SSQ_{reg}	$= \sum_{i=1}^{n} ((\hat{y}_i - \bar{y})^2)$
SSQ_{res} (or SSE)	$= \sum_{i=1}^{n} ((y_i - \hat{y}_i)^2)$
SSQ_x	$= \sum_{i=1}^{n} ((x_i - \bar{x})^2)$
F	$= SSQ_{reg}/(SSQ_{res}/df)$
RMSE	$= \sqrt{SSQ_{res}/df}$
CV(RMSE)	$= RMSE/\bar{y}$
R-Squared	$= SSQ_{reg}/SSQ_{total}$
R-Squared	$= 1 - (SSQ_{res}/SSQ_{total})$
Adjusted R-Squared	$= 1 - \left((1 - R^2) * \frac{n-1}{n-p-1}\right)$
Net Determination Bias	$= \sum_{i=1}^{n} (y_i - \hat{y}_i) \Big/ \sum_{i=1}^{n} y_i$
Confidence Half-Interval	$= t\text{-}statistic * SE * \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{SSQ_x}}$
Prediction Half-Interval	$= t\text{-}statistic * SE * \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SSQ_x}}$

Table 5-2: Microsoft Excel Functions for Regression Model Statistics

Regression Model Statistic	Microsoft Excel Function	Excel LINEST (Where Applicable)
n	*= COUNT(XVals)*	
p	2	
df	*= n-p*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 4,2)*
Y_{avg}	*= AVERAGE(Yvals)*	
X_{avg}	*= AVERAGE(XVals)*	
SSQ_{total}	*= DEVSQ(Yvals)*	
SSQ_{reg}	*= DEVSQ(YvalsCalc)*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 5,1)*
SSQ_{res} (or SSE)	*= SUM((Yvals-YvalsCalc)^2)*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 5,2)*
SSQ_x	*= DEVSQ(XVals)*	
F	*= DEVSQ(YvalsCalc)/(SUM((Yvals-YvalsCalc)^2)/(n-p))*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 4,1)*
RMSE	*= SQRT(SUM((Yvals-YvalsCalc)^2)/(n-p))*	
CV(RMSE)	*= SQRT(SUM((Yvals-YvalsCalc) ^2)/(n-p))/AVERAGE(Yvals)*	
R-Squared	*= RSQ(Yvals,XVals)*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 3,1)*
R-Squared	*= RSQ(Yvals,XVals)*	
Adjusted R-Squared	*= 1-((1-RSQ(Yvals,XVals))*((n-1)/ (n-p-1)))*	
Net Determination Bias	*= SUM(Yvals-YvalsCalc)/SUM(Yvals)*	
Confidence Half-Interval	Evaluated at each x	
Prediction Half-Interval	Evaluated at each x	

Table 5-3: Definitions of Coefficient Statistics

Coefficient Statistic	Equation or Definition
Confidence Level	Input required probability that the coefficient is not zero
***t*-Statistic, Critical**	From table
Intercept	$= \bar{y} - slope * \bar{x}$
Slope	$= \frac{\sum_{i=1}^{n}\left((x_i - \bar{x}) * (y_i - \bar{y})\right)}{\sum_{i=1}^{n}((x_i - \bar{x})^2)}$
Standard Error of Intercept	$= \sqrt{\frac{SSQ_{res}}{n-p} * \left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)}$
Standard Error of Slope	$= \sqrt{SSQ_{res}/(n-p)/SSQ_x}$
***t*-Statistic for Intercept**	$= Intercept/(Standard\ Error\ of\ Intercept)$

Coefficient Statistic	Equation or Definition
***t*-Statistic for Slope**	$= Slope/(Standard\ Error\ of\ Slope)$
***p*-Value for Intercept**	—
***p*-Value for Slope**	—

Table 5-4: Microsoft Excel Functions for Coefficient Statistics

Coefficient Statistic	Microsoft Excel Function	Excel LINEST (Where Applicable)
Confidence Level	0.95	
***t*-Statistic, Critical**	*= T.INV.2T((1-ConfLvl)/2,n-p)*	
Intercept	*= INTERCEPT(Yvals,XVals)*	
Slope	*= SLOPE(Yvals,XVals)*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 1,2)*
Standard Error of Intercept	*= STEYX(Yvals,XVals)*SQRT (1/n+Xavg^2/DEVSQ(XVals))*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 1,1)*
Standard Error of Slope	*= STEYX(Yvals,XVals)*SQRT (1/DEVSQ(XVals))*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 2,2)*
***t*-Statistic for Intercept**	*= (INTERCEPT(Yvals,XVals))/ (STEYX(Yvals,XVals)*SQRT (1/n+Xavg^2/DEVSQ(XVals)))*	*= INDEX(LINEST(Yvals,XVals, TRUE,TRUE), 2,1)*
***t*-Statistic for Slope**	*= (SLOPE(Yvals,XVals))/ (STEYX(Yvals,XVals)*SQRT (1/DEVSQ(XVals)))*	
***p*-Value for Intercept**	*= TDIST(ABS(INTERCEPT (Yvals,XVals))/(STEYX(Yvals, XVals)*SQRT(1/n+Xavg^2/ DEVSQ(XVals))),n-p,2)*	
***p*-Value for Slope**	*= TDIST(ABS(SLOPE(Yvals,XVals))/ (STEYX(Yvals,XVals)*SQRT (1/DEVSQ(XVals))),n-p,2)*	

6. Example

6.1. Use of Monthly Billing Data in a 2-Parameter Model to Evaluate Whether It Will Make a Satisfactory Baseline

Regression is commonly used to analyze monthly utility data. It is best applied to a package of measures whose total savings is a relatively high percentage of the building's baseline energy use. It is important to remember that the energy use of buildings is typically dependent on weather. More specifically, it can be dependent on the demand for cooling and heating. This is because energy usage is usually higher when it is either very cold (heaters) or very hot (AC units), since the temperature is far from the balance point.

In cases where only cooling or only heating is present or relevant, a simple 2-parameter (straight-line) regression is often satisfactory.

Consider the case of schools in the Northwest, especially on the west side of the Cascade Mountains. Many schools do not have cooling, and although cooling is not generally needed during the school year, heating is. Therefore, a model of energy use versus *average ambient temperature* or *heating degree-days* (HDD) may be appropriate.

Usually, degree-days are better than average-ambient-temperatures. An average temperature may indicate little need for heating or cooling if it is near the balance point for the building. However, a moderate average temperature can be made up of a series of cool temperatures and a series of warm temperatures. During the times of cool temperatures, heating is needed. Therefore, depending on climate, a better fit will typically be found by using degree-days. On the west side of the Cascades in the Northwest, winter temperatures may be relatively constant over a day, and almost always below a school building's balance point, so the greatest difference between degree-days and average temperature will be found in the spring and fall months.

The following analysis estimates the baseline for the electricity use of a group of modular classrooms heated by heat pumps. The planned measure is a web-enabled programmable thermostat. Prior similar projects have shown savings exceeding 45% of a building's baseline energy use.

The available data are the monthly electricity energy use (kWh) and ambient temperature during the billing period. There are 24 months of data to be used for the baseline. The data to be used for the regression will be normalized to *average kWh per day* in each billing period and *average heating degree-days* per day in each billing period. The base temperature for heating degree-days in this example is 65º F. (See Section 6.2 for a discussion of heating degree-days.)

The relevant equation is for a common 2-parameter *ordinary least squares regression*:

- $Y = \beta_1 + \beta_2 X_1$

  where: Y = electricity use per day in the billing period

  β_1 = *y-intercept* – electricity use (kWh-per-day) for a day with zero heating degree-days

  β_2 = *slope* – how much the energy use increases for a day as the temperature decreases below 65° F (kWh-per-day per heating degree-day)

  X_1 = average heating degree-days per day in the billing period

Table 6-1 provides the data for the project.

Table 6-1: Example Data for Classroom Heat Pump Project

End of Billing Period	Billing Period Duration in Days	Billed Usage kWh	HDD in Billing Period
09/24/2007	30	6,080	113
10/24/2007	30	7,330	311
11/21/2007	28	7,470	463
12/19/2007	28	10,000	669
01/23/2008	35	11,480	877
02/25/2008	33	11,420	782
03/26/2008	30	9,970	560
04/24/2008	29	7,840	561
05/20/2008	26	6,800	265
06/21/2008	32	5,980	268
07/23/2008	32	4,310	73
08/22/2008	30	3,330	57

The consumption and heating degree-days are standardized by the number of days in the billing period (Table 6-2).

Table 6-2: Data Standardized by Days in the Billing Period

End of Billing Period	Billing Period Duration in Days	Billed Usage kWh	HDD in Billing Period	Average kWh per Day in Billing Period	Average HDD per Day in Billing Period
09/24/2007	30	6,080	113	202.7	3.7
10/24/2007	30	7,330	311	244.3	10.4
11/21/2007	28	7,470	463	266.8	16.5
12/19/2007	28	10,000	669	357.1	23.9

End of Billing Period	Billing Period Duration in Days	Billed Usage kWh	HDD in Billing Period	Average kWh per Day in Billing Period	Average HDD per Day in Billing Period
01/23/2008	35	11,480	877	328.0	25.1
02/25/2008	33	11,420	782	346.1	23.7
03/26/2008	30	9,970	560	332.3	18.7
04/24/2008	29	7,840	561	270.3	19.4
05/20/2008	26	6,800	265	261.5	10.2
06/21/2008	32	5,980	268	186.9	8.4
07/23/2008	32	4,310	73	134.7	2.3
08/22/2008	30	3,330	57	111.0	1.9

Table 6-3 provides the *Microsoft Excel* formulas for the regression. Note that *p* in the term (*n-p*) refers to the number of parameters, which is two for this simple linear regression.

Table 6-3: Microsoft Excel Formulas for the Regression

Output	Formula
R-squared	*= RSQ(Yvals,XVals)*
Number of Baseline Points, n	*= COUNT(YVals)*
CV(RMSE)	*= SQRT(SUM((Yvals-YvalsCalc)^2)/(n-p))/AVERAGE(Yvals)*
Intercept at HDD=0	*= INTERCEPT(Yvals,XVals)*
Slope	*= SLOPE(Yvals,XVals)*

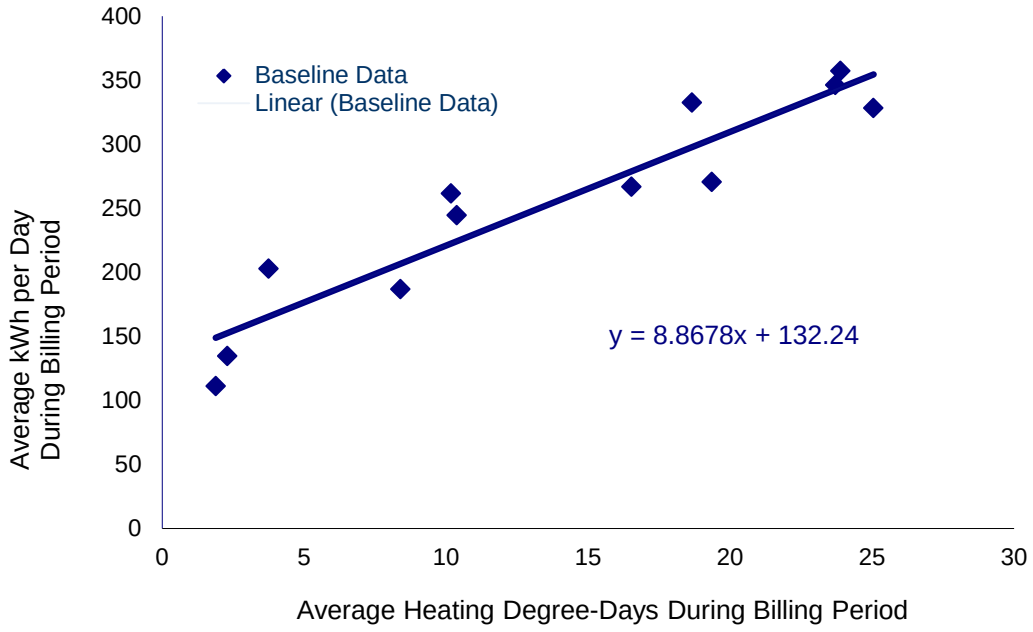
Table 6-4 provides the *Excel* output:

Table 6-4: Microsoft Excel Output for Example Model

Output	Data
R-squared	= 0.879
Number of Baseline Points, n	= 12
CV(RMSE)	= 11.7%
Intercept at HDD=0	= 132.24
Slope	= 8.8678

Figure 6-1 shows the data graphed, with the regression equation and line included.

Figure 6-1: Baseline Electricity Use vs. Heating Degree-Days



Next, the uncertainty needs to be calculated. The input confidence level used to calculate the *t*-statistic will be 90%. The *t*-statistic will be used to get the confidence intervals, evaluated at each value of *X*. To calculate the *t*-statistic, some intermediate calculations need to be made, as shown in Table 6-5. In this table, *p* is the probability that the dependent variable is not significantly related to the independent variable.

Table 6-5: Microsoft Excel Formulas for the Fit Statistics

Output	Formula
Standard Error	*= STEYX(Yvals,XVals)*
Standard Error – Percent of Average	*= STEYX(Yvals,XVals) / AvgY*
Critical *t*-Statistic	*= TINV(1-ConfLvl,n-p)*
Sum of Squares of Differences: *X-avg(X)*	*= DEVSQ(XVals)*
Standard Deviation of the Residuals	*= STDEV(Residuals)*
***t*-Statistic**	*= CONFIDENCE(1-ConfLvl,STDEV(Residuals),n)*
***p*-Value**	*= TDIST(ABS(t_statistic),n-p,2)*

Table 6-6 provides the *Excel* outputs for the goodness-of-fit statistics.

Table 6-6: Microsoft Excel Output for Example Fit Statistics

Output	Data
Standard Error	= 29.69
Standard Error – Percent of Average	= 11.7%
Number of Baseline Points, n	= 12
Critical *t*-Statistic	= 12
Sum of Squares of Differences: *X-avg(X)*	= 1.81
Standard Deviation of the Residuals	= 818
***t*-Statistic**	= 28.3
***p*-Value**	= 13.44

Below is the equation for calculating the confidence intervals for the regression:

- $\Delta Y_{confidence} = \pm$*(t-statistic)*STEYX(Yvals,XVals)*SQRT(1/n+(X-xAvg)^2/DEVSQ(XVals))*

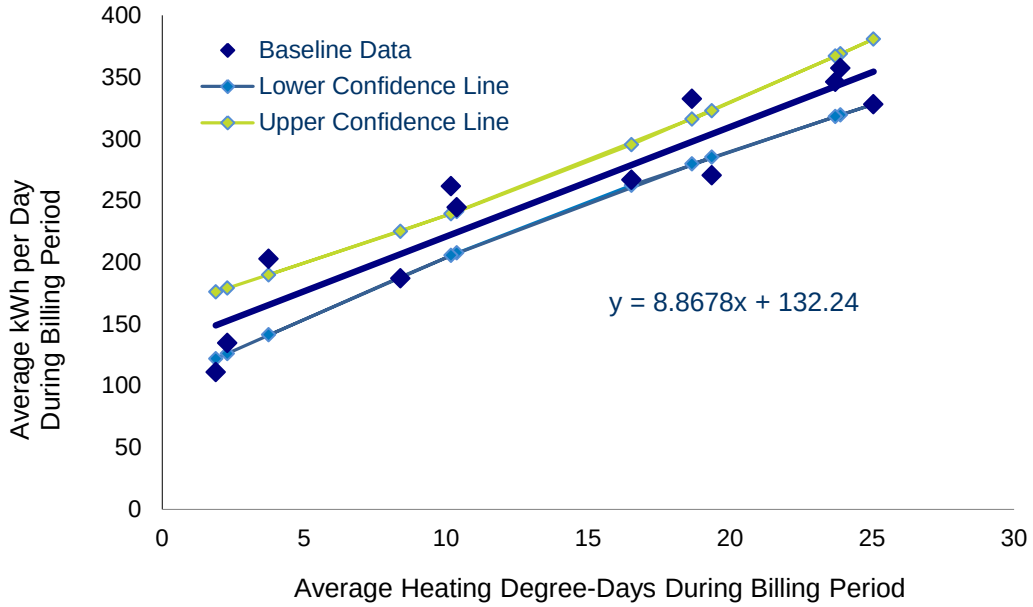
Table 6-7 provides the spreadsheet output, including the estimates for the confidence intervals of the regression. *Min Modeled* is the modeled value minus the confidence half-interval. *Max Modeled* is the modeled value plus the confidence half-interval.

Table 6-7: Example Model Estimates

Average HDD per Day in Billing Period	Average kWh per Day in Billing Period	Modeled kWh per Day	Residual	90% Confidence Half-Interval	Minimum Modeled kWh per Day	Maximum Modeled kWh per Day
3.7	202.7	165.5	37.2	24.3	141.2	189.8
10.4	244.3	224.2	20.1	16.7	207.5	241.0
16.5	266.8	278.8	-12.0	16.4	262.4	295.3
23.9	357.1	344.1	13.1	24.7	319.4	368.8
25.1	328.0	354.4	-26.4	26.5	327.9	380.8
23.7	346.1	342.5	3.6	24.5	318.0	366.9
18.7	332.3	297.7	34.6	18.2	279.6	315.9
19.4	270.3	303.9	-33.6	18.9	285.1	322.8
10.2	261.5	222.4	39.1	16.9	205.6	239.3
8.4	186.9	206.6	-19.7	18.4	188.2	225.1
2.3	134.7	152.6	-17.9	26.5	126.1	179.0
1.9	111.0	149.0	-38.0	27.1	121.9	176.1
Total	**3,041.8**	**3,041.8**	**0.0**	**258.9**	**2,782.8**	**3,300.7**

Figure 6-2 provides the scatter chart again, including the lines of 90% confidence intervals.

Figure 6-2: Baseline Electricity Use vs. Heating Degree-Days with Confidence Intervals



Note that the regression appears to reproduce the baseline totals. However, these values are for the average kWh-per-day, not for the total energy use over the year. Yet each point does not represent the same number of days; consequently, the best approach would have been to use a weighted regression. Because a weighted regression was not used, the model's bias should be checked.

To complete the model and check the bias, the modeled values for kWh-per-day are multiplied by the number of days in the billing period. The actual kWh values are reproduced in Table 6-8 for comparison with the modeled values.

Table 6-8: Example Model Estimates with Actual Observations

Average HDD per Day in Billing Period	Average kWh per Day in Billing Period	Modeled kWh per Day	Residual	90% Confidence Half-Interval	Minimum Modeled kWh per Day	Maximum Modeled kWh per Day	Actual kWh	Modeled kWh
3.7	202.7	165.5	37.2	24.3	141.2	189.8	**6,080**	**4,964**
10.4	244.3	224.2	20.1	16.7	207.5	241.0	**7,330**	**6,727**
16.5	266.8	278.8	-12.0	16.4	262.4	295.3	**7,470**	**7,807**
23.9	357.1	344.1	13.1	24.7	319.4	368.8	**10,000**	**9,634**
25.1	328.0	354.4	-26.4	26.5	327.9	380.8	**11,480**	**12,403**
23.7	346.1	342.5	3.6	24.5	318.0	366.9	**11,420**	**11,301**
18.7	332.3	297.7	34.6	18.2	279.6	315.9	**9,970**	**8,932**
19.4	270.3	303.9	-33.6	18.9	285.1	322.8	**7,840**	**8,814**
10.2	261.5	222.4	39.1	16.9	205.6	239.3	**6,800**	**5,783**
8.4	186.9	206.6	-19.7	18.4	188.2	225.1	**5,980**	**6,612**
2.3	134.7	152.6	-17.9	26.5	126.1	179.0	**4,310**	**4,883**
1.9	111.0	149.0	-38.0	27.1	121.9	176.1	**3,330**	**4,469**
Total	**3,041.8**	**3,041.8**	**0.0**	**258.9**	**2,782.8**	**3,300.7**	**92,010**	**92,331**

So, what is the bias in the model?

- **Net Determination Bias Error (NDBE):** $NDBE = 100 * \frac{\sum_i (E_i - \hat{E}_i)}{\sum_i E_i}$

  $NDBE$ = (92,331 – 92,010) / 92,331

  $NDBE$ = 0.3%

The model predicts 0.3% higher energy use than the actual data.

ASHRAE Guideline 14 does not accept a model with bias >0.005%, so this model would be rejected. However, the uncertainty in the model is much, much greater than the bias, and the savings are expected to be much, much greater than the uncertainty. Thus, this model is acceptable:

- **Modeled Uncertainty** = ± (92,331 – 84,436) / 92,331 = ± 8.6%.

The expected energy savings for this measure is at least 45%. Since the uncertainty is low relative to the expected savings, this baseline model would be acceptable for projecting energy use under post-implementation conditions and could be used in the calculation of energy savings.

6.2. Background on Heating and Cooling Degree-Days (HDD and CDD)

Heating degree-days are a measure of how much cold weather there is in a specific period. The average daily temperature is determined for each day. The average temperature is then compared to a *base* temperature (often 65º F). If the average temperature (when only daily data are available, typically the average of the daily high and the daily low) is 55º F for a day, and the base is 65º F, then that day contributes 10 HDD to the period. The HDD for each day in the period (typically a calendar month or a utility billing period) are summed to create a single data point for the month. If the temperature difference for a day is negative, it is recorded as 0.

- $HDD_n = \sum_{i}^{n} \left(T_{base} - T_i\right)^{+}$

Note that while HDD and CDD are often reported with a base or balance point of 65º F, results can often be improved by experimenting with different base temperatures. The base temperature should generally be the average temperature at which the building does not require any heating or cooling – the balance point temperature. For most commercial buildings, this temperature will typically be between 55° and 60º F, depending on building size, operating schedule, and other parameters. If regression models are created separately for occupied and unoccupied periods, the balance point temperature will be different for each: for the occupied period, it may be near 55º F, and for the unoccupied period it may be near 65º F.

7. Minimum Reporting Requirements

This document is a reference guide, a companion to the M&V protocols. Below are the minimum reporting requirements for the use of regressions within protocols. The overall M&V approach should be described according to the minimum reporting requirements of the protocol used. Please see the protocols for minimum reporting requirements.

These are the essential reporting requirements for regressions within an M&V plan and verification report:

- **Data:** variables, interval of observation – such as monthly, number of observations, or length of measurement period
- **Model:** the proposed or final model and alternative models proposed or tested (the verification report should include estimated model parameters)
- **Model Statistics:** statistics for assessing goodness of fit (proposed and, in the verification report, calculated statistics for final model)
- **Discussion**: why the final model was selected or weaknesses of the alternative models tested

8. References and Resources

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Appendix: Glossary of Statistical Terms

This Glossary provides definitions for the statistical terms used in this *Regression Reference Guide*. Additional M&V terms are defined in the companion document *Glossary for M&V: Reference Guide.*

Accuracy: An indication of how close the measured value is to the true value of the quantity in question. Accuracy is not the same as precision.

Adjusted R-square ($\overline{R}^2$): A modification of R^2 that adjusts for the number of independent variables (explanatory terms) in a model. The adjusted R^2 only increases if the additional independent variables improve the model more than by random chance. It is calculated by taking R^2 and dividing it by the associated degrees of freedom. Or as described below:

$$\bar{R}^2 = 1 - \frac{MSE}{MST}$$

Autocollinearity: The serial correlation over time of predictor values in a time series model. To calculate autocollinearity, R-squared is first calculated for the correlation between the residuals and the residuals for the prior period. The autocorrelation coefficient ρ is then the square root of this value.[17] Autocollinearity is calculated as:

$$\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$$

Categorical Variables: Variables that have discrete values and are not continuous. Categorical variables include things like daytype (weekday or weekend, or day of week), occupancy (occupied or unoccupied), and equipment status (on or off). For example, occupancy (occupied or unoccupied) is a categorical variable, while number of occupants is a continuous variable.

Coefficient of Variation (CV): An indication of how much variability or randomness there is with any given data set. It quantifies variation within the population relative to the average and is dimensionless. The larger it is, the more variation there is in the population relative to the average. It is calculated as the ratio of the standard deviation to the average:

$$CV = \frac{\sigma}{\bar{x}}$$

[17] Note, the English spelling of the Greek letter ρ is *rho*, not to be confused with "p."

Coefficient of Variation of the Root-Mean Squared Error [CV(RMSE)]: A measure that describes how much variation or randomness there is between the data and the model, calculated by dividing the root-mean squared error (RMSE) by the average y-value. It is calculated as:

$$CV(RMSE) = \frac{1}{\bar{y}}\left[\frac{\sum(y_i - \hat{y}_i)^2}{(n-p)}\right]^{1/2}$$

Confidence Interval: A range of uncertainty expected to contain the true value within a specified probability. The probability is referred to as the *confidence level*.

Confidence Level: A population parameter used to indicate the reliability of a statistical estimate. The confidence interval expresses the assurance (probability) that given correct model selection, the true value of interest resides within the proportion expressed by the confidence interval.

Continuous Variables: Variables that are numeric and can have any value within the range of encountered data (that is, measurable things such as energy usage or ambient temperature).

Dependent Variable: The variable that changes in relationship to alterations of the independent variable. In energy efficiency, energy usage is typically treated as the dependent variable, responsive to the manipulation of conditions (independent variables).

Homoscedasticity: (Also known as *Homogeneity of Variance.*) Within linear regression, this means that the variance of the dependent values around the regression line is constant for all values of the independent variable.

Independent Variable: Also termed an *explanatory* or *exogenous variable*; a factor that is expected to have a measurable impact on the dependent, or outcome variable (such energy use of a system or facility).

Mean: The most widely used measure of the central tendency of a series of observations. The Mean ($\overline{Y}$) is determined by summing the individual observations (Y_i) and dividing by the total number of observations (n), as follows:

$$\overline{Y} = \frac{1}{n}\sum Y_i$$

Mean Bias Error (MBE): The Mean Bias Error is an indication of overall bias in a regression model. Positive MBE indicates that regression estimates tend to overstate the actual values. It is calculated as:

$$MBE = \frac{\sum(y_i - \hat{y}_i)}{n}$$

Mean Model: (Also known as a *Single Parameter Model*.) A model that estimates the mean of the dependent variable.

Multicollinearity: A statistical occurrence where two or more predictor variables in a multiple regression model are highly correlated (there are exact linear relationships between two or more explanatory variables). Allowing multicollinearity in a model can lead to incorrect inferences from the model.

Net Bias: Where there exists net bias, modeled or predicted energy usage will differ from actual energy usage for the period examined.

Net Determination Bias Error (NDBE): The percentage error in the energy use predicted by the model compared to the actual energy use. See *Normalized Mean Bias Error*.

$$NDBE = 100 * \frac{\sum_i (E_i - \hat{E}_i)}{\sum_i E_i}$$

Normal Distribution: A continuous and symmetric population distribution in which the frequency of occurrence decreases exponentially as values deviate from the mean (or central) value. In a regression equation, the distribution of errors (residuals) at a given value of *x* is a normal distribution and the mean of residuals is zero. It is also referred to as a *Gaussian* or *bell curve*.

Normalized Mean Bias Error (NMBE): Similar to *Net Determination Bias* but adjusted for the number of parameters in the model. The Normalized Mean Bias Error is an indication of overall bias in a regression model. Positive MBE indicates that regression estimates tend to overstate the actual values. It is calculated as:

$$NMBE = 100 * \frac{\sum_i (E_i - \hat{E}_i)}{(n-p) * \bar{E}}$$

Ordinary Least Squares (OLS): A mathematical procedure to solve for the set of coefficients that minimize the sum of the squared differences between the raw data and the fitted linear trend. OLS is the most common form of regression modeling and the default approach in most software packages.

Outliers: Data points that do not conform to the typical distribution. Graphically, an outlier appears to deviate markedly from other members of the same sample.

Overspecified Model: A model with added independent variables that are not statistically significant or are possibly correlated with other independent variables.

***p*-value:** The probability that a coefficient or dependent variable is not related to the independent variable. Small *p*-values, then, indicate that the independent variable or coefficient is a significant (important) predictor of the dependent variable in a regression model. The *p*-value is an alternate way of evaluating the *t*-statistic for the significance of a regression coefficient and is expressed as a probability.

Precision: The indication of the closeness of agreement among repeated measurements; a measure of the repeatability of a process. Any precision statement about a measured value must include a confidence level. A precision of 10% at 90% confidence means that we are 90% certain the measured values are drawn from samples that represent the population and that the "true" value is within ±10% of the measured value. Because precision does not account for bias or instrumentation error, it is an indicator of predicted accuracy only given the proper design of a study or experiment.

R-Squared (R^2): (Also known as the *Coefficient of Determination*.) R^2 is the measure of how well future outcomes are likely to be predicted by the model. It illustrates how well the independent variables explain variation in the dependent variable. R^2 values range from 0 (indicating none of the variation in the dependent variable is associated with variation in any of the independent variables) to 1 (indicating all the variation in the dependent variable is associated with variation in the independent variables, a "perfect fit" of the regression line to the data). It is calculated as:

$$R^2 = 1 - \frac{SS_{error}}{SS_{total}}$$

Regression Analysis: A mathematical technique that extracts parameters from a set of data to describe the correlation relationship of measured independent variables and dependent variables.

Regression Model: A mathematical model based on statistical analysis where the dependent variable is regressed on the independent variables which are said to determine its value. In so doing, the relationship between the variables is estimated from the data used. A simple linear regression is calculated as:

$$y_i = \beta_0 + \beta_i x_i + \varepsilon_i, \text{ where } i = 1,\ldots, n$$

Reliability: When used in energy evaluation, refers to the likelihood that the observations can be replicated.

Residual: The difference between the predicted and actual value of the dependent variable. In other words, whether a point is above or below the regression line is a matter of chance and is not influenced by whether another point is above or below the line. Estimated by subtracting the data from the sample mean:

$$\hat{\varepsilon} = X_i - \overline{X}$$

Root Mean Squared Error (RMSE): An indicator of the scatter, or random variability, in the data, and hence is an average of how much an actual *y*-value differs from the predicted *y*-value. It is the standard deviation of errors of prediction about the regression line. The RMSE is calculated as:

$$RMSE(\hat{\theta}) = \sqrt{MSE(\hat{\theta})} = \sqrt{E((\hat{\theta} - \theta)^2)}$$

Standard Deviation (s): The square root of the variance, which brings the variability measure back to the units of the data. (With variance units in kWh^2, the standard deviation units are kWh.) The sample standard deviation (*s*) is calculated as:

$$s = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{(n-1)}}$$

Standard Error (SE): An estimate of the standard deviation of the coefficient. For simple linear regression, it is calculated separately for the slope and intercept: there is a *standard error of the intercept* and *standard error of the slope*. SE is calculated as:

$$SE_x = \frac{s}{\sqrt{n}}$$

Standard Error of the Coefficient: Similar to the *RMSE* but calculated for a single coefficient rather than the complete model. This measures the degree to which the coefficient estimate may change if the full process was to be repeated.

***t*-statistic:** A measure of the probability that the value (or difference between two values) is statistically valid. The calculated *t*-statistic can be compared to critical *t*-values from a *t*-table. The *t*-statistic is inversely related to the *p*-value; a high *t*-statistic (t>2) indicates a low probability that random chance has introduced an erroneous result. Within regression, the *t*-statistic is a measure of the significance for each coefficient (and, therefore, of each independent variable) in the model. The larger the *t*-statistic, the more significant the coefficient is to the estimation of the dependent variable. The t-statistic is calculated as:

$$t_{\hat{\beta}} = \frac{\hat{\beta} - \beta_0}{s.e.(\hat{\beta})}$$

Uncertainty: The range or interval of doubt surrounding a measured or calculated value within which the true value is expected to fall within some stated degree of confidence. Uncertainty in regression analysis can come from multiple sources, including *measurement uncertainty* and *regression uncertainty*.

Variance (S^2): A measure of the average distance between each of a set of data points and their mean value, and it is equal to the sum of the squares of the deviation from the mean value, or the square of the standard deviation. Variance is computed as follows:

$$S^2 = \frac{\sum(Y_i - \bar{Y})^2}{n-1}$$

Weighted Regression: A form of regression used when individual data points are weighted to represent more data than other points. An example is billing-period analysis, where billing periods may have different numbers of days and billing periods with more days are adjusted upward in weight relative to periods with fewer days. (Also, a form of regression used when data do not have equal weight in a model because error is not expected to be constant across all observations.)

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